MATHEMATICS

GRADE 8

TEACHERS GUIDE

AUTHORS

1. **Tolosa Diriba (B.Sc)**
2. **Mequaninit Teshome (B.Sc)**
3. **Afework Awlachew(M.Sc)**

Evaluators

1. **Rodas Diriba**
2. **Mustefa Kedir**
3. **Gesit Assefa**

Developed by Addis Ababa Educational Bureau and Adapted by South Nations Nationalities and People Regional Education Bureau.

ADAPTED BY:

NATHAN LABISO LANGANO (MSc)

CURRICULUM DIRECTORATE

FIRST EDITION, 2014 E.C

ACTIVE LEARNING AND CONTINUOUS ASSESSMENT REQUIRED

Dear Mathematics teachers! For generation the technique of teaching mathematics at any level was dominated by what is commonly called the direct instruction. That is students are given the exact tools and formulas they need to solve a certain mathematical problem, sometimes without a clear explanation as to why, and they are told to do certain steps in a certain order and in turn are expected to do them as such at all times. This leaves little room for solving varying types of problems. It can also lead to misconceptions and students may not again the full understanding of the concept that is being taught.

You just sit back for a while and try to think the most common activities that you, as a mathematics teacher, are being in the class.

Either you explain (lecture) the new topic to them, and expect your students to remember and use the contents of this new topic or you demonstrate with examples how a particular kind of problem is solved and students routinely imitate these steps and procedures to find answers to a great number of similar mathematical problems.

But this method of teaching revealed little or nothing of the meaning behind the mathematical process the students were imitating.

We may think that teaching is telling students something, and learning occurs if students remember it. But research reveals that teaching is not 'pouring' information into students' brain and expecting them to process it and apply it correctly later.

Most educationalists agree that learning is an active meaning-making process and students will learn best by trying to make sense of something on their own with the teacher as a guide to help them along the way. This is the central idea of the concept Active Learning. Active learning, as the name suggests, is a process whereby learners are actively engaged (involved) in the learning process, rather than "passively" absorbing lectures. Students are rather encouraged to think, solve problems, do activities carefully selected by the teacher, answer questions, formulate questions of their own, discuss, explain, debate, or brainstorm, explore and discover, work cooperatively in groups to solve problems and workout projects.

The design of the teaching materials (students' textbooks and teachers' guide) for mathematics envisages active learning to do dominantly use. With this strategy, we feel that you should be in a position to help students understand the concepts through relevant, meaningful and concrete activities. The activities should be carried out by students to explore the world of mathematics, to learn, to discover and to develop interest in the subject. Though it is your role to exploit the opportunity of using active learning at an optimal level, for the sake of helping you get an insight, we recommended that you do the following as frequently as possible during your teaching.

- Engage your students in more relevant and meaningful activities than just listening.

- Include learning materials having examples that relate to students' life, so that they can make sense of the information.
- Let students be involved in dialog, debate, writing, and problem solving, as well as higher-order thinking, e.g., analysis, synthesis, evaluation.
- Encourage students' critical thinking and inquiry by asking them thoughtful, open-ended questions, and encourage them to ask questions to each other.
- Have the habit of asking learners to apply the information in a practical situation. This facilitates personal interpretation and relevance.
- Guide them to arrive at an understanding of a new mathematical concept, formula, theorem, rule or any generalization, by themselves. You may realize this by giving them an activity in which students sequentially uncover layers of mathematical information one step at a time and discover new mathematics.
- Select assignments and projects that should allow learners to choose meaningful activities to help them apply and personalize the information. These need to help students undertake initiatives, discover mathematical results and even design new experiments to verify results.
- Let them frequently work in peers or groups. Working with other learners gives learners real-life experience of working in a group, and allows them to use their metacognitive skills. Learners will also be able to use the strengths of other learners, and to learn from others. When assigning learners for group work membership, it is advisable if it is based on the expertise level and learning style of individual group members, so that individual team members can benefit from one another's strengths.

In general, if mathematics is to develop creative and imaginative mathematical minds, you must overhaul your traditional methods of presentation to the more active and participatory strategies and provide learning opportunities that allow your students to be actively involved in the learning process. While students are engaged with activities, group discussions, projects, presentations and many others they need to be continuously assessed.

Continuous Assessment

You know that continuous assessment is an integral part of the teaching learning process. Continuous assessment is the periodic and systematic method of assessing and evaluating a person's attribute and performance. Information collected from continuous behavioural change of students will help teachers to better understand their strengths and weakness in addition to providing a comprehensive picture of each student over a period of time. Continuous assessment will afford student to readily see his/her development pattern through the data. It will also help to strengthen the parent teacher relationship and

collaboration. It is an on-going process more than giving a test or exam frequently and recording the marks.

Continuous assessment enables you to assess a wide range of learning competencies and behaviours using a variety of instruments some of which are:

- ✓ Tests/ quizzes (written, oral or practical)
- ✓ Class room discussions, exercises, assignments or group works.
- ✓ Projects
- ✓ Observations
- ✓ Interview
- ✓ Group discussions
- ✓ Questionnaires

Different competencies may require different assessment techniques and instruments.

For example, oral questions and interviews may serve to assess listening and speaking abilities. They also help to assess whether or not students are paying attention, and whether they can correctly express ideas. You can use oral questions and interviews to ask students to restate a definition, note or theorem, etc. questionnaires, observations and discussions can help to assess the interest, participation and attitudes of a student. Written tests/exams can also help to assess student's ability to read, to do and correctly write answers for questions.

When to Assess

Continuous and instruction are integrated in three different time frames namely, Pre-instruction, During-instruction and Post-instruction. To highlight each briefly

1. **Pre-instruction assessment**

This is to assess what students luck to start a lesson. Hence you should start a lesson by using opportunities to fill any observed gap. If students do well in the pre-instruction assessment, then you can begin instructing the lesson. Otherwise, you may need to revise important concepts.

The following are some suggestions to perform or make use of pre-instruction assessment.

i) Assess whether or not students have the prerequisite knowledge and skill to be successful, through different approaches.
ii) Make your teaching strategies motivating.
iii) Plan how you form groups and how to give marks.
iv) Create interest on students to learn the lesson.

2. **Assessment during instruction:**

This is an assessment during the course of instruction rather than before it is started or after it is completed. The following are some of the strategies you may use to assess during instruction.

i) Observe and monitor students' learning.
ii) Check that students understand the lesson. You may use varying approaches such as oral questions, asking students to do their work on the board, stimulate discussion, etc.
iii) Identify which students need extra help and which students should be left alone.
iv) Ask a balanced type of exercise problems according to the students' ability, help weaker students and give additional exercise for fast students.
v) Monitor how class works and groups discussions are conducted.

3. **Post instruction Assessment:**

This is an assessment after instruction is completed. It is conducted usually for the purpose of documenting the marks and checking whether competencies are achieved. Based on the results students scored, you can decide whether or not there is anything the class did not understand because of which you may revise some of the lessons or there is something you need to adjust on the approach of teaching. This also help you analyse whether or not the results really reflect what students know and what they can do, and decide how to treat the next lesson.

Forming and managing groups

You can form groups through various approaches: mixed ability, similar ability, gender or other social factors such as socioeconomic factors. When you form groups, however, care need to be taken in that you should monitor their effort. For example, if students are grouped by mixed ability the following problems may happen.

1. Mixed ability grouping may hold back high-ability students. Here, you should give enrichment activities for high ability students.
2. High ability students and low ability students might form a teacher-student relationship and exclude the medium ability students from group discussion. In this case you should group medium ability students together.

When you assign group work, the work might be divided among the group members, who work individually. Then the members get together to integrate, summarize and present their finding as a group project. Your role is to facilitate investigation and maintain cooperative effort.

UNIT 1

RATIONAL NUMBERS

Total allotted period: 30 periods

Introduction

In the previous grade the students have learnt about the set of natural numbers, whole numbers, and integers and their basic properties. In this unit we introduce the set of numbers which contains the other set of numbers, which is rational number. The unit gives much emphasis to the definition of rational numbers, representation of rational numbers on a number line, relationship among N, W, Z, and Q, absolute value of rational numbers, comparing rational numbers, ordering rational numbers, addition of rational numbers, subtraction of rational numbers, multiplication of rational numbers, division of rational numbers, application in sharing among friends, and application in calculating interest and loans.

Unit Outcomes

At the end of this unit, students will be able to:

- Define and represent rational numbers as fractions.
- Show the relationship among W, $\mathbb{Z}$ and $\mathbb{Q}$.
- Order rational numbers
- Solve problems involving Addition, Subtraction, Multiplication and Division of rational numbers.
- Apply Rational Numbers to solve practical problems.
- Aware the four operations as they relate to Rational Numbers.

Suggested teaching Aids in unit 1

You know that students learn in a variety of different ways. Some are visually oriented and more inclined to acquire information from photographs or videos. Others do best when they hear instructions rather than read them. Teachers use teaching aids to provide these different ways of learning. Therefore, you can present these topics using Venn diagrams, number lines, thermometer, fractional bars and grids model to understand about rational numbers.

1.1 The Concept of Rational Numbers

Period Allotted: 6 periods

Competencies:

At the end of this sub-unit, students should:

- Describe the concept of Rational Numbers practically
- Express Rational Numbers as fractions.

1.1.1. Representation of Rational Numbers

Introduction:

To begin, it is better to motivate the students by giving an insight of the course, the unit. How they will be manipulating specific kinds of numbers, which are related to their real life activities. After that, describe how mathematicians classify numbers and what kinds of major classifications exist. After these discussions you are continue to discuss the group activity. The purpose of group activity 1.1 is more to rehearse the lower grade concepts that are related to the unit. Let each student do Group Work 1.1 with group and you should give much emphasis on the ways developing numbers. The answer to Group work 1.1 is as follows.

Teaching notes

The purpose of the teaching notes is to provide the teacher information to use activities, opening problems and group-works to motivate and guide students rather than lecturing.

Students are expected to have some background on the concept of natural numbers, whole numbers, and integers from the previous grades. For the purpose of revision, you can ask different questions like the following:

1) Group the following numbers are natural number, whole number, integer or not
   a) $\frac{12}{4}$ b) $\frac{-36}{3}$ c) $\frac{0}{7}$ d) $\frac{37}{13}$
2) Between what consecutive integers the following rational numbers exist?
   a) $\frac{5}{2}$ b) $\frac{-3}{2}$ c) $\frac{3}{4}$ d) $\frac{-5}{7}$
3) Change the following improper fractions to mixed fractions.
   a) $\frac{12}{7}$ b) $\frac{25}{6}$ c) $\frac{19}{7}$
4) Change the following mixed fractions to improper fractions.
   a) $3\frac{2}{3}$ b) $2\frac{4}{7}$
5) Represent the following rational numbers on a number line.
   a) $\frac{7}{6}$ b) $\frac{-8}{3}$ c) $\frac{3}{5}$

Answer to Group Work 1.1

1a: 1, 2, 3, 4, 5, 6, 7, 8, and 9

b: 0

c: …-3, -2, -1

2a: integers

3: any number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, is called a rational number.

4: Given: total number of toys = 5

Total amount in Birr = 22.

Let x be the amount spent for each pet. Then

$5x = 22$

$\frac{1}{5} \times 5x = 22 \times \frac{1}{5}$

$x = \frac{22}{5}$. This means the amount spent for each toy is Birr 4 and 40 cents.

Following these discussion, you are continue to discuss the idea of fraction, its type and define what a rational number is? And relate the definition with fractions and decimals. You can use similar examples to the examples given in the student textbook. The possible answers to exercise 1.1 are as follows.

Answer to Exercise 1.1

1: d, a number line between 1 and 3.5 divides inton5 equal parts, each part represents $\frac{1}{2}$. Then when we add $\frac{1}{2}$, starting from 1 until we get point B, that is 2

2: a) between 0 and 1

b) between 1 and 2

c) between -1 and 0

d) between -2 and -1

5) Divide the numerator by the denominator. The quotient uses as the whole number part, the remainder uses as the numerator and the denominator uses as the denominator of the proper fraction.

a) 32÷5 = 6 remainder 2

Therefore, $\frac{32}{5} = 6\frac{2}{5}$

b) 27÷10 = 2 remainder 7

Therefore, $-\frac{27}{10} = -2\frac{7}{10}$

c) 7÷ 3 = 2 remainder 1

Therefore, $\frac{7}{3} = 2\frac{1}{3}$

4) First determine between what consecutive integers the given rational numbers exist.

a) $\frac{5}{6}$ is between 0 and 1 on a number line, and divide a number line between 0 and 1 into 6 equal parts, the fifth part represents $\frac{5}{6}$ of the 6 equal parts.

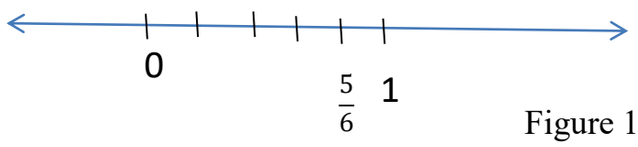


Figure 1.1

b) $\frac{3}{5}$ is between 0 and 1, and divide a number line between 0 and 1 into 5 equal parts, the third part represents $\frac{3}{5}$ of the 5 equal parts.

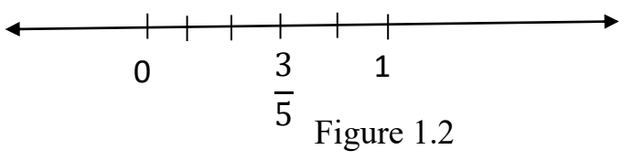


Figure 1.2

c) $-\frac{5}{6}$ is between – 1 and 0, and divide a number line between – 1 and 0 into 6 equal parts. The fifth part represents $-\frac{5}{6}$ of the 6 equal parts.

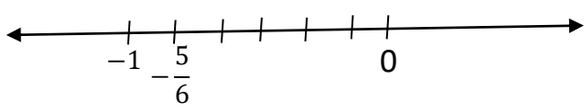


Figure 1.3

d) $-\frac{8}{5}$ is improper fraction, then first change into mixed fraction.

$-\frac{8}{5} = -1\frac{3}{5}$ and it is between -2 and -1 on a number line. Divide a number line between -2 and -1 into 5 equal parts. The third part represents $-\frac{8}{5}$.

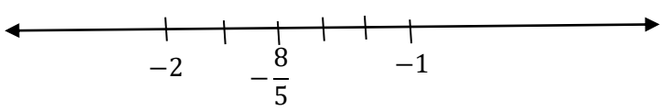


Figure 1.4

e) $2\frac{2}{5}$ is between 2 and 3. Divide a number line between 2 and 3 into 5 equal parts. The second part represents $2\frac{2}{5}$.

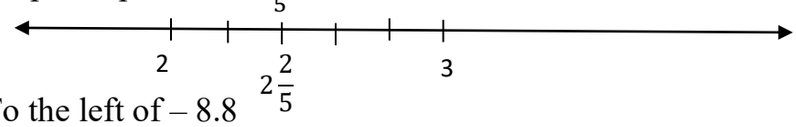


Figure 1.5

5) To the left of – 8.8

6) a) $-\frac{4}{5}$ b) $\frac{2}{5}$ c) $-2\frac{3}{5}$ d) $3\frac{4}{7}$

Teachers' are strongly advised to discuss and workout difficult questions as far as possible. The class should not be teacher centred. Following these exercise, you are continue the sub-section.

Dear teacher, please give the following questions for slow learners and fast learners.

For slow learners

1) Describe Natural numbers, Whole numbers, and Integers by your own words.

2) Group the following numbers are Natural numbers, Whole numbers, and Integers or not.

a) $\frac{15}{3}$ b) $\frac{13}{12}$ c) $\frac{-18}{6}$ $\frac{0}{13}$

For fast learners

1) Describe how to represent a rational number on a number line.
2) Represent the following rational numbers on a number line.

a) $\frac{35}{12}$ b) $\frac{-21}{8}$

1.1.2. Relationship Among W, $\mathbb{Z}$ and $\mathbb{Q}$

Competencies:

At the end of this sub – unit students should be able to:

- Describe the relationship among the sets W, $\mathbb{Z}$ and $\mathbb{Q}$

The teacher may begin the subtopic by encourage the students to answer the question like:

a) Solomon says the number 0 belongs only to the set of rational numbers. Explain his error.

b) Is it possible for a number to be a rational number that is not an integer but is a whole number? Explain.

These questions help the teacher to understand the level of the student's critical thinking and critique reasoning. After giving a brief summary about the relationship among $\mathbb{N}$, W, $\mathbb{Z}$, and $\mathbb{Q}$ and you should allow enough time for the student to discuss the relationship among $\mathbb{N}$, W, $\mathbb{Z}$, and $\mathbb{Q}$. Give exercise 1.2 as a home work in order to analyse their knowledge of the relationship among $\mathbb{N}$, W, $\mathbb{Z}$, and $\mathbb{Q}$. And can give marks and record it.

Answers to Exercise 1.2

1) 0 is not only rational number. It is an integer and also whole number.

2) a) True b) True c) False

1.1.3. Absolute Value of Rational numbers

Competency: -

After completing this sub-section, students should be able to:

- Determine the absolute value of a rational numbers.

Dear teacher before discussing to section 1.1.3 directly, ask your students orally to answer activity 1.1. This activity is used to introduce the concept of absolute value.

Answer for Activity 1.1

The distance from 0 and 5 is 5 units and from 0 and – 5 is also 5 units, the only difference is the direction with related to 0.

Here you strongly advised to deal with absolute value of a rational number as **the concept is new,** otherwise students will face problems in understanding meaning of absolute value and solving absolute value equations. By using simpler examples, guide your students work continuously and compare it with the competency to determine whether the student has achieved the minimum required level. So give enough time for the students to discuss exercise 1.3 and come up with their own solutions. You should also give feedback for their questions and answers timely and explicitly.

Answers to exercise 1.3

1:

x	-7	$\frac{3}{5}$	0	$-2\frac{5}{9}$ or $2\frac{5}{9}$	$-\frac{2}{3}$
$\lvert x \rvert$	7	$\frac{3}{5}$	0	$2\frac{5}{9}$	$\frac{2}{3}$

2) **a**. 3.5 and – 3.5 b. $\frac{4}{7}$ and $-\frac{4}{7}$

c. $\frac{11}{6}$ and $-\frac{11}{6}$ d. $3\frac{2}{5}$ and $-3\frac{2}{5}$

3a) 10 b) 12 c) $3\frac{2}{5}$ d) 3 e) $\frac{1}{13}$

4) a) $5x - \lvert x-3 \rvert, x = -5$

$5 \times (-5) - \lvert -5-3 \rvert$

$= -25 - \lvert -8 \rvert$

$= -25 - 8$

$= -33$

b) $\lvert x \rvert - x + 9, x = 3$

$= \lvert 3 \rvert - 3 + 9$

$= 3 + 6$

$= 9$

c) $\lvert x+y \rvert - \lvert x \rvert, x = -3$ and $y = 6$

$= \lvert -3+6 \rvert - \lvert -3 \rvert$

$= \lvert 3 \rvert - 3$

$= 3 - 3$

$= 0$

d) $\lvert x \rvert + \lvert y \rvert$,$x = 5$ and $y = -10$

$= |5| + |-10|$

$= 5+10$

$= 15$

e) $-3|x+6|, x = -5$

$= -3|-5+6|$

$= -3|1|$

$= -3$

f) $\frac{|x|-|5y|}{|x+y|}$, $x = 4$ and $y = 8$

$= \frac{|4|-|5\times 8|}{|4+8|}$

$= \frac{|4|-|40|}{|12|}$

$= \frac{-36}{12}$

$= -3$

5 a) $x = 8$ or $x = -8$

b) $x = \frac{3}{5}$ or $x = -\frac{3}{5}$

6 a) $x = 6$ or $x = -6$

b) $x = -6$ or $x = 0$

c) $3-2|x-5| = 9$

$-2|x-5| = 9-3$

$-2|x-5| = 6$

$|x-5| = -3$

No solution, because absolute value cannot be negative

d) $|5x-3| = \frac{5}{2}$

$5x-3 = \frac{5}{2}$ or $5x-3 = -\frac{5}{2}$

$5x = \frac{5}{2}+3$ $5x = -\frac{5}{2}+3$

$5x = \frac{5}{2}+\frac{6}{2}$ $5x = -\frac{5}{2}+\frac{6}{2}$

$5x = \frac{11}{2}$ $5x = \frac{1}{2}$

$x = \frac{11}{10}$ $5 = \frac{1}{10}$

e) $3|x-5| = 3\frac{2}{3}$

$|x-5| = \frac{11}{9}$

$x-5=\frac{11}{9}$ or $x-5=-\frac{11}{9}$

$x=\frac{11}{9}+5$ $x=-\frac{11}{9}+5$

$x=\frac{56}{9}$ $x=\frac{34}{9}$

Assessment

Dear teacher you are strongly advised to guide students by grouping them in two groups such as slow learners, and fast learners. Simple problems and difficult problems are respectively to be solved to slow learners and fast learners. using such a steep the slow learners will be pushed up to fast learners, you can give the following additional questions.

For slow learners

1) What is the distance between 0 and – 17?
2) Find the opposite of the following rational numbers.
   a) – 18 b) $\frac{11}{5}$ c) $\frac{0}{7}$
   b)
3) Solve the following absolute value equations.
   a) $|x|=9$
   b) $|x-3|=7$
   c) $|x+4|=3$

For fast learners

1) Evaluate $\frac{|2x|-|3x-y|}{|2x-3y|}$, $x=5$ and $y=3$
2) Solve the following absolute value equations.
   a) $3+2|x+5|=7$
   b) $3|x-2|=8$
   c) $4-3|x+7|=3$

1.2. Comparing and Ordering Rational numbers

Period Allotted: 6 periods

Competency:

At the end of this subtopic students should:

❖ Compare and order Rational Numbers

In the previous sections students have studied about rational numbers and how to locate rational numbers on the number line. You may start the lesson by a quick revision on

definition of rational numbers, locating rational numbers on the number line and guide students to extend this idea for comparing rational numbers using number line. The sub-topic is divided into two subtopics. The first subtopic is Comparing rational numbers and the second subtopic is ordering rational numbers. To assess students background on comparison, you can give activity 1.2 to do with groups and encourage each group to come up with answers and reasons.

1.2.1. Comparing Rational numbers

Students understand the given lesson differently at different level. Therefore, in order to ensure that all students can fully participate in their learning, you may give various techniques such as class work, group discussion and quiz. These helps to check students how much they achieved. You can start this lesson by giving different examples.

Example: Compare the following pair of numbers by using the symbols

<, > or =.

a) 13 ____ 21 e) 11.23____ 8.67

b) $\frac{17}{8}$ _____ $\frac{13}{8}$ f) $\frac{12}{7}$ _____ $\frac{15}{10}$

c) $3\frac{2}{7}$ _____ $3\frac{4}{9}$

Answer to Activity 1.2

a) < b) > c) < d) =

After completing the given activity, examples, and short note, you can give exercise 1.4 as a class work or homework. Finally, proper feedback must be given for each question.

Answers to exercise 1.4

1) a) True b) False c) False d) False
e) True f) True g) False h) True

2) a) < b) = c) > d) < e) = f) >

3) a) 3.5 b) $\frac{5}{7}$ c) $2\frac{5}{7}$ d) -9 e) -1.5

4) a) < b) < c) > d) >

1.2.2. Ordering Rational Numbers

Before you discuss the order of rational numbers, first you may remind the students how to compare the fractions like $\frac{5}{7}$ and $\frac{6}{7}$, $\frac{5}{7}$ and $\frac{5}{9}$, and the decimals 0.35 and 0. 371. At this moment, you can observe the students understanding and making generalizations on comparing two rational numbers. Then you can explain to the student what is meant by ordering. After giving a brief explanation, you should allow enough time to discuss

ordering of rational numbers. You give exercise 1.5 as a home work in order to analyse their knowledge of ordering of rational numbers.

Answers to exercise 1.5

1 a) $-5\frac{2}{3} < \frac{3}{25} < \frac{4}{9} < \frac{23}{5} < \frac{11}{7}$

b) $-12.51 < 5.24 < 6.75 < 8.13$

c) $\frac{5}{13} < \frac{11}{9} < 3.92 < 4.73 < 4\frac{6}{7}$ [Convert each number to the same notation, either in fraction form or decimal form and then compare]

2 a) $23.86 > 15.02 > 13.72 > 13.05$

b) $2\frac{7}{9} > 2\frac{4}{9} > \frac{21}{12} > \frac{9}{7} > \frac{13}{16}$

c) $4.23 > 3\frac{5}{6} > 3.73 > \frac{18}{5} > 3.2$

3 a) Rahel's plant

b) $-2.2 < -1\frac{7}{10} < 1.7 < 3\frac{1}{4}$

Assessment

Dear teacher, at this time you have analyse the understanding of your students. Please assess your students. Give different problems to compare and order rational numbers using <, >, or = and check their work.

For slow learners

1) Compare the following rational numbers by using the symbols <, > or = .
   a) 7.25 _____ 9.13
   b) $\frac{3}{7}$ _____ $\frac{2}{5}$
   c) $2\frac{1}{3}$ _____ $3\frac{1}{2}$
   d) 2.75 ____ $2\frac{3}{4}$
2) Arrange the following rational numbers in:
   i) Ascending order
   ii) Descending order
   a) 12.32, 35.74, 17.4, 21.78
   b) $\frac{12}{7}, \frac{9}{5}, \frac{7}{3}, 2\frac{3}{4}$
   c) $3.42, 3\frac{2}{3}, \frac{13}{4}, 4.38$

For fast learners

1) Compare the following rational numbers by using the symbols <,>or =
   a) $\left|2\frac{3}{7}\right|$ ------ $\left|-2\frac{3}{7}\right|$
   b) 13.67 ------- $\frac{37}{3}$
2) Arrange the following rational numbers in:
   i) Ascending order

ii) Descending order

a) 15.67 , $12\frac{3}{7}$, $\frac{23}{4}$, 21.54

1.3. Operations and properties of Rational Numbers

Period Allotted: 14periods

Competencies:

After completing this sub-topic, students should:

- ❖ Add rational numbers.
- ❖ Subtract rational numbers.
- ❖ Multiply rational numbers.
- ❖ Divide rational numbers.

Use the opening problem to assess the background and the initiation the students have to continue the topic. It is better to use performance assessment methods such as engaging students in debate on real life problem like the opening problem given here.

The intention of the assessment is to provide a tool for the identification of students who are experiencing major difficulty. Such an assessment is effective for planning a remedial program. Assessment should provide the teacher with a valuable profile of each student's strengths and weakness, and so enable the teacher to direct her/his teaching at the identified weaknesses.

Opening questions

For integers and decimals, we can rely on our calculators to add, subtract, multiply, and divide them. However, basic calculators can't give us answers in fractions. Therefore, we need to be able to perform fractions by hand.

Dear teacher please give the following expressions to discuss how to perform the given operations.

1) $\frac{6}{10}+\frac{3}{10}$
2) $\frac{3}{4}+\frac{9}{7}$
3) $\frac{12}{9}-\frac{2}{3}$
4) $\frac{4}{9}\times\frac{3}{5}$
5) $\frac{7}{9}\div\frac{5}{4}$

1.3.1. Addition of rational numbers

You can start this sub-topic by recalling addition of integers using a number line. After that divide students into groups and let them discuss by stating and showing the **rule for addition** of two rational numbers using examples.

- If the signs of the addends are different
  - i) Take the sign of the addend with the greater absolute value.
  - ii) Take the absolute values of both numbers and subtract the addend with smaller absolute value from the addend with greater absolute value.
- If both rational numbers are negative
  - i) Decide (put) the sign first.
  - ii) Take the sum of the absolute values of the addend.

And ask your students to find addition of rational numbers using

a) The above properties of addition
b) Fractional bars and
c) Using scientific calculator.

For instance, the sum of $\frac{3}{8} + \frac{2}{8}$ using fractional bars.

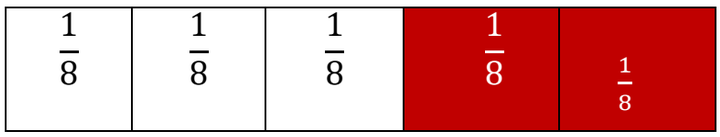


Figure 1.6

Answer to Activity 1.3

1) The students know how to represent rational numbers on a number line.
   a) Move 6 units from zero to the right side and back 3 units from 6, then we get 3.

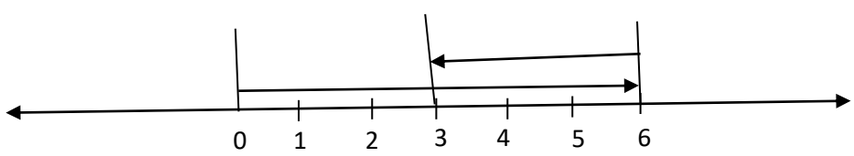


Figure 1.7

Therefore, $6 + (-3) = 3$

b) move 9 units from zero to the left side of zero and back 5 units to zero, then we get -4.

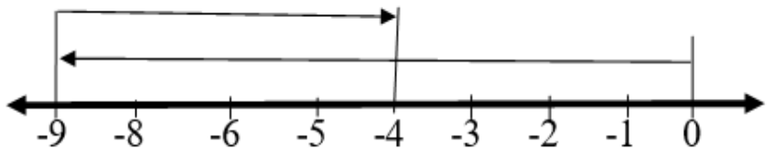


Figure 1.8

Therefore, - 9+5 = -4

c) move 2 units from zero to the right side of zero and back 4 units from 2, then we get -2

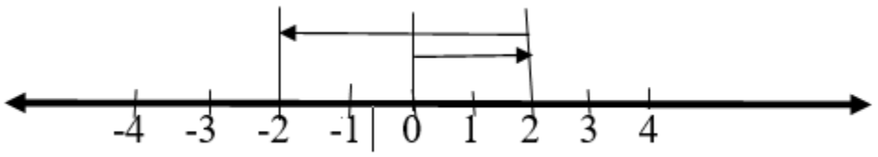


Figure 1.9

Therefore, $2 + (-4) = -2$

d) Move 6 units from zero to the right side and back 6 units, then we get o.

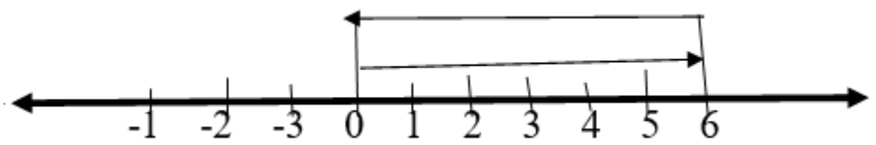


Figure 1.20

Therefore, $6 + (-6) = 0$

2) Divide the fraction bar in to 6 equal parts and shade 3 of them with grey colour and 2 of them with red colour as shown below.

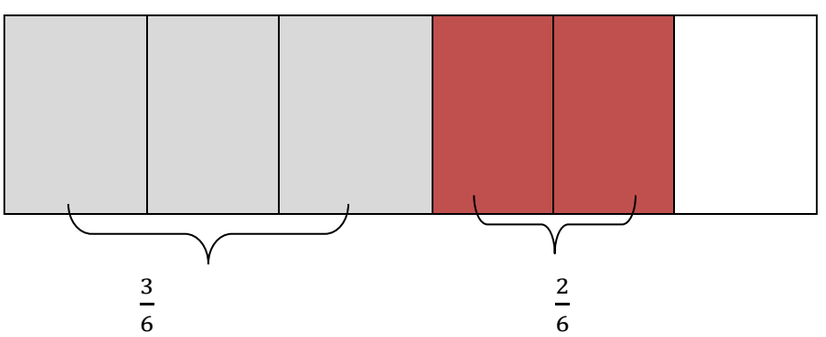


Figure 1.21

How many parts of the fraction bar is shaded? 5, This implies that $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

Answer to exercise 1.6

1.a. $\frac{13}{5} + \frac{21}{5} = \frac{13+21}{5} = \frac{34}{5}$

b. $\frac{5}{6} + \frac{3}{8}$

first change to fractions with the same denominator

LCM (6,8) is 24

Then $\frac{5}{6}\times\frac{4}{4}+\frac{3}{8}\times\frac{3}{3}=\frac{20}{24}+\frac{9}{24}=\frac{29}{24}$

c. $\frac{3}{5}+2\frac{3}{5}=\frac{3}{5}+\frac{2\times5+3}{5}=\frac{3}{5}+\frac{13}{5}=\frac{16}{5}$

d. $2\frac{1}{3}+\frac{3}{8}+3\frac{5}{6}=\frac{7}{3}+\frac{3}{8}+\frac{23}{6}=\frac{56}{24}+\frac{9}{24}+\frac{92}{24}=\frac{157}{24}$

e. $\left|-\frac{2}{5}+\frac{3}{8}\right|+\left|\frac{4}{7}+\frac{2}{7}\right|=\left|-\frac{16}{40}+\frac{15}{40}\right|+\left|\frac{4+2}{7}\right|$

$=\left|\frac{-16+15}{40}\right|+\left|\frac{6}{7}\right|$

$=\left|\frac{-1}{40}\right|+\left|\frac{6}{7}\right|$

$=\frac{-1}{40}+\frac{6}{7}$

$=\frac{-7}{280}+\frac{240}{280}$

$=\frac{-7+240}{280}$

$=\frac{233}{280}$

3) $23.72^{o}c + 3.23^{o}c = 26.95^{o}c$

Assessment

Continuous assessment addresses various strategies that teachers can use in order to ensure that all students in their class can fully participate in meaningful discussion.

Dear teacher you can also give class activities, group discussion, assignments, exercise problem and quiz or test for assessing students learning. Finally divide the students into two groups such as slow learners and fast learners. And give simple problems and difficult problems respectively for slow learners and fast learners.

Slow learners

1) Perform the following operation

a) $\frac{7}{9}+\frac{13}{9}$

b) $\frac{11}{4}+\frac{8}{12}$

c) $2\frac{1}{3}+\frac{5}{7}$

2) What number should be added to $-\frac{7}{9}$ so as to get $-\frac{4}{3}$?

Fast learners

1. From a rope 17 m long, two pieces of lengths $\frac{13}{5}$ m and $\frac{34}{7}$ m are cut off. What is the length of the remaining rope?
2. The sum of two rational numbers is $\frac{67}{12}$. If one of the number is $\frac{16}{3}$. then what is the other number?

1.3.2 Subtraction Rational Numbers

First give chance for the students to recall subtraction of integers. It is as a motivation to start subtraction of rational numbers. Please teacher encourage the students to express subtraction of rational numbers as addition of the opposite of the rational number to be subtracted.

Example: $7-5=7+(-5)$

Advise the students to apply the rules learnt for addition of rational numbers to find the difference of rational numbers.

Answer to Activity 1.4

1) Yes
2) Yes

Answer to Exercise 1.7

1) a) $4\frac{5}{6}-2\frac{3}{4}=\frac{29}{6}-\frac{11}{4}=\frac{58}{12}-\frac{33}{12}=\frac{58-33}{12}=\frac{25}{12}$

b) $-5.3-3.45=-8.75$

c) $\frac{6}{13}-\left|-\frac{7}{13}\right|=\frac{6}{13}-\frac{7}{13}=-\frac{1}{13}$

d) $\left|-\frac{5}{7}\right|-\left|\frac{3}{4}\right|=\frac{5}{7}-\frac{3}{4}=\frac{20}{28}-\frac{21}{28}=-\frac{1}{28}$

e) $-32.24-\left|-32.24\right|=-32.24-32.24=-64.48$

f) $-3\frac{2}{5}-2\frac{3}{7}=-\frac{17}{5}-\frac{17}{7}=-\frac{119}{35}-\frac{85}{35}=\frac{-119-85}{35}=\frac{-204}{35}=-5\frac{29}{35}$

2) a) $\frac{9}{4}-(\frac{2}{7}+5)=\frac{9}{4}-(\frac{2}{7}+\frac{35}{7})=\frac{9}{4}-(\frac{37}{7})=\frac{63}{28}-\frac{138}{28}=\frac{63-138}{28}=-\frac{75}{28}$

b) $15-(-7-\frac{4}{9})=15-(-\frac{63}{9}-\frac{4}{9})=15-(-\frac{67}{9})=15+\frac{67}{9}=\frac{202}{9}$

3) $23-(\frac{12}{7}+\frac{7}{4})=23-\frac{133}{28}=\frac{644-133}{28}=\frac{511}{28}=18\frac{7}{28}$

Therefore, the length of the remaining rope is 18.25 meter.

4) $\frac{58}{3}-(\frac{18}{7}+\frac{11}{9})=\frac{58}{3}-\frac{162+77}{63}=\frac{58}{3}-\frac{239}{63}=\frac{1218-239}{63}=\frac{979}{63}=15\frac{34}{63}$

Therefore, bananas weighing is $15\frac{34}{63}$

Assessment

Always remember to consider the minimum learning competencies that is expected of the students. As part of your assessment technique, it mainly indicates that your students should determine the difference of rational numbers.

Dear teacher, please give the following questions for slow and fast learners.

For slow learners

Perform the following operations

a) 13.56 – 10.4

b) $\frac{16}{13} - \frac{5}{13}$

c) $\frac{7}{9} - \frac{3}{5}$

d) $23.63 - \frac{9}{7}$

e) $\left(\frac{3}{4} + \frac{-2}{5}\right) + \ -\frac{7}{10}$

f) $-3 + (-\frac{3}{4} + \ -\frac{5}{3})$

Fast learners

1) Perform the following operation

a) $-\frac{4}{10} + 2\frac{7}{8}$

b) $-\frac{13}{9} - 3\frac{5}{6}$

c) $3\frac{4}{7} - 2\frac{3}{5}$

d) $-4\frac{2}{5} - 2\frac{7}{9}$

2) A drum full of rice weighs $\frac{234}{5}$ kg. If the empty drum weighs $\frac{64}{6}$ kg, find the weight of rice in the drum.

3) A basket contains tree types of fruits weighing $\frac{71}{3}$ kg in all. If $\frac{43}{7}$ kg of these be apples, $\frac{13}{4}$ kg be oranges and the rest pears. What is the weight of the pears in the basket?

4) Is the difference of two rational numbers a rational number?

1.3.3 Multiplication of Rational Numbers

You can start this sub- topic by giving a chance for the students to revise multiplication of rational numbers and rules of multiplication of integers.

i. The product of two integers with different signs

a. Decide the sign of the product, it is ‘ – ‘

b. Take the product of the absolute value of the numbers

c. Put the sign (-) in front of the product.

ii. The product of two negative integers

a. Decide the sign of the product, it is '+'

b. Take the product of the absolute value of the numbers.

After revise these rules Come to multiplication of rational numbers and the rules learnt for multiplication of integers can be applied to find the product of rational numbers.

Encourage the students to discuss activity 1.4 with in groups. You should allow enough time to share their ideas.

Answer to Activity 1.4

a. $\frac{5}{7}\times\frac{8}{11}=\frac{5\times8}{7\times11}=\frac{40}{77}$

b. $-\frac{9}{7}\times\frac{4}{5}=\frac{-9\times4}{7\times5}=-\frac{36}{35}$

c. $\frac{2}{3}\times(\frac{5}{9}\times\left(-\frac{3}{5}\right))=\frac{2}{3}\times(\frac{5\times-3}{9\times5})=\frac{2}{3}\times(-\frac{15}{45})=\frac{2\times(-15)}{3\times45}=-\frac{30}{135}=-\frac{2}{9}$ d. $3\frac{2}{5}\times 2\frac{4}{7}=\frac{17}{5}\times\frac{18}{7}=\frac{17\times18}{5\times7}=\frac{306}{35}$

e. $-\frac{3}{8}\times(-\frac{2}{3})=\frac{-3\times(-2)}{8\times3}=\frac{1}{4}$

Answer to Exercise 1.8

1) a. $\frac{15}{32}$ b. $-\frac{9}{28}$ c. $\frac{24}{35}$ d. $\frac{182}{15}$ e. $-\frac{5}{4}$

2) $1250\times\frac{23}{6}=625\times\frac{23}{3}=\frac{14375}{3}=4791\frac{2}{3}$

Therefore, An airplane covers $4791\frac{2}{3}$ km in $\frac{23}{6}$ hours.

1.3.4 Division of Rational Numbers

Before discussing division of rational numbers revise that the reciprocal of a rational number. For any rational number $\frac{a}{b}$, $\frac{a}{b}\times\frac{b}{a}=1$, then $\frac{b}{a}$ is the reciprocal of $\frac{a}{b}$.

Now come to division of rational numbers and encourage students to discuss division as the product of the dividend and reciprocal of the divisor.

Example: $\frac{3}{4}\div\frac{2}{5}=\frac{3}{4}\times\frac{5}{2}$

Encourage students to find the quotient of rational numbers using grids to model.

To determine the sign of the quotient of rational numbers you can apply the rules that you have learnt for multiplication of rational numbers.

Answer to Activity 1.6

1) $12\div\frac{3}{4}=12\times\frac{4}{3}=16$

2) $3\frac{3}{5}\div\frac{2}{5}=\frac{18}{5}\times\frac{5}{2}=9$

3) Divide each 4 grids in to three equal parts. Shade 3 grids and $\frac{1}{3}$ of a fourth grid to represent $3\frac{1}{3}$ and divide the shaded grids in to equal groups of 2.

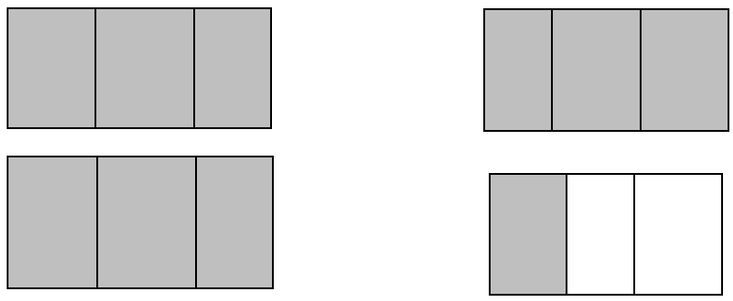


Figure 1.2.2

Thus, There are 5 groups of $\frac{2}{3}$.

Therefore, $3\frac{1}{3} \div \frac{2}{3} = 5$

Answer to Exercise 1.9

1) a) $\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3} = \frac{5}{6}$

b) $-\frac{3}{5} \div \frac{6}{7} = -\frac{3}{5} \times \frac{7}{6} = -\frac{7}{10}$

c) $-\frac{8}{9} \div (-\frac{5}{3}) = -\frac{8}{9} \times (-\frac{3}{5}) = \frac{8}{15}$

d) $2\frac{3}{5} \div (-\frac{4}{3}) = \frac{13}{5} \times (-\frac{3}{4}) = -\frac{39}{20}$

2) To find the number of bags, we have to divide $\frac{3}{4}$ by $\frac{3}{8}$.

$\frac{3}{4} \div \frac{3}{8} = \frac{24}{12} = 2$

Assessment

You should assess each student's work continuously over the whole period and discuss how to divide a rational numbers and some rules to determine the sign of the quotient. Finally give different exercise problem for slow learners and fast learners.

For slow learners

1) Determine the quotient.

a) $\frac{6}{7} \div \frac{5}{14}$

b) $-\frac{2}{9} \div \frac{6}{12}$

c) $2\frac{3}{4} \div \frac{5}{9}$

2) The cost of 8 meters of cloth is Birr $\frac{567}{5}$. What is the cost of one meter of cloth?

For fast learners

1) Determine the quotient
   a) $-\frac{13}{7} \div \frac{32}{15}$
   b) $-3\frac{6}{11} \div -4\frac{5}{7}$
   c) $4.56 \div \frac{3}{8}$
2) A cord of length $\frac{143}{2}$ m has been cut into 26 pieces of equal length. What is the length of each piece?
3) The product of two rational numbers is $\frac{48}{5}$. If one of the rational number is $\frac{66}{7}$, find the other rational number.

1.4 Real-life Applications of Rational Numbers

Period Allotted: 4 periods

Competencies:

After completing this sub-topic, students should:

- Solve application of Rational Number.

1.4.1. Application in Sharing something among friends

You can start this sub-topic by divide the students with in groups and discuss about application of rational numbers in your day to day activities. After the student's discussion you conclude their ideas.

Note: The application of rational numbers is prevalent in our daily lives. They are a part of our daily calculations like following the recipes, determine shopping discounts, calculating prices, assessing the most cost-effective size of products, preparing budgets, savings, reading financial statements, etc.

Example: Dividing a birthday cake among five friend; each friend will receive one-fifth part of the cake, which can be represented using a rational number?

1.4.2 Application in calculating Interest and loans

Before start this sub-topic encourage students to revise about simple interest and how to calculate simple interest from grade 7 mathematics lessons.

Answer to Exercise 1.9

1. $I = p \times R \times T = 120 \times 10\% \times 5$

   $= 120 \times \frac{10}{100} \times 5 = 60$

   Therefore, the simple interest is Birr 60

2. I = p×R×T from this you can calculate the principal as:

   $P = \frac{I}{RT} = \frac{637}{\frac{7}{100}\times 2} = 637 \times \frac{50}{7} = 4550$

   Therefore, the principal is Birr 4550.

3. P = Birr 6000

   A = Birr 7500

   T = 5 month = $\frac{5}{12}$ year

   To calculate the interest rate of a loan,

   I = PRT but

   I = A−P

   7500 – 6000

   = 1500

   Since R = $\frac{I}{PT}$

   $= \frac{1500}{6000\times\frac{5}{12}}$

   $= \frac{15}{25}$

   $= \frac{3}{5}$

   Therefore, the interest rate is 60%.

Assessment

Dear teacher asses your students' based on the following concepts on rational numbers:

- ✓ Give different rational numbers to represent on a number line.
- ✓ Give different exercise to show the relationship among ℕ, W, ℤ, and ℚ.
- ✓ Give different exercise to describe the absolute value of rational numbers and represent on a number line.
- ✓ Give different exercise to evaluating equations involving absolute values.
- ✓ Give different exercise to comparing and ordering rational numbers in order to face real-life problems.
- ✓ Give different exercise on addition of rational numbers.
- ✓ Give different exercise to apply properties of addition.
- ✓ Give different exercise on subtraction of rational numbers.
- ✓ Give different exercise on multiplication of rational numbers and its properties.
- ✓ Give different exercise on division of rational numbers.
- ✓ Give different exercise problems to evaluating interest and loan.

You should check the students' performance in each assessment and give feedback. Finally, based on the performance of your students:

Give class work.

Give assignment.

Give project work.

Give tests.

Give the following exercise for fast learners.

1. There are 36 students in a class and $\frac{2}{3}$ of them are female students. How many students are female? And male?
2. An industrial factory makes $4\frac{2}{3}$ yards of chain every minute. The cost of each yard of chain is Birr 6.25. what is the total cost for all chain produced in 10 minutes?
3. What is the term of loan where Birr 600 was borrowed at a simple interest of 8% and the interest paid on the loan was Birr 156?

Answer to Miscellaneous Exercise 1

1. a. false c. true
   b. true d. true
2. a) $x = 7$ or $x = -7$
   b) $x - 5 = 4$ or $x - 5 = -4$
   $x = 9$ $x = 1$
   c) $2x + 3 = 5$ or $2x + 3 = -5$
   $2x = 2$ $2x = -8$
   $X = 1$ or $x = -4$
   d) $3x - 4 = 3$ or $3x - 4 = -3$
   $3x = 7$ $3x = 1$
   $X = \frac{7}{3}$ $x = \frac{1}{3}$
   e) $5 + 3|x - 3| = 2$
   $3|x - 3| = 2 - 5$
   $|x - 3| = -1$, the absolute value cannot be negative
   Therefore, the equation has no solution.
3) $x = -8$, $y = 4$

$$\frac{3|x-5|-|4y|}{|x+y|} = \frac{3|-8-5|-|4\times4|}{|-8+4|}$$

$$= \frac{3|-13|-|16|}{|-4|}$$

$$= \frac{3(13)-16}{4}$$

$$= \frac{23}{4}$$

4) a) $\frac{3}{4}+\frac{9}{7}+2\frac{3}{5}=\frac{105}{140}+\frac{180}{140}+\frac{364}{140}=\frac{649}{140}=4\frac{89}{140}$

b) $-2\frac{1}{3}+1\frac{4}{7}=-\frac{7}{3}+\frac{11}{7}=-\frac{49}{21}+\frac{33}{21}=\frac{-49+33}{21}=-\frac{16}{21}$

c) $3.35+2\frac{3}{7}=\frac{67}{20}+\frac{17}{7}=\frac{469}{140}+\frac{340}{140}=\frac{809}{140}$

5) a) $\frac{2}{7}-\frac{3}{8}=\frac{16}{56}-\frac{21}{56}=-\frac{5}{56}$

b) $-2\frac{6}{7}-\frac{13}{9}=-\frac{20}{7}-\frac{13}{9}=-\frac{180}{63}-\frac{92}{63}=-\frac{272}{63}$

6) a) $2\frac{3}{7}\times\frac{1}{4}=\frac{17}{7}\times\frac{1}{4}=\frac{17}{28}$

b) $2.34\times\frac{7}{6}=\frac{117}{50}\times\frac{7}{6}=\frac{819}{300}=\frac{273}{100}$

c) $-3\frac{3}{5}\times1\frac{2}{3}\times(-\frac{3}{4})=-\frac{18}{5}\times\frac{5}{8}\times(-\frac{3}{4})=\frac{27}{16}$

7) a) $\frac{3}{4}\div(-\frac{2}{7})=\frac{3}{4}\times(-\frac{7}{2})=-\frac{21}{8}$

b) $-3\frac{5}{8}\div2\frac{3}{10}=-\frac{29}{8}\div\frac{23}{10}=-\frac{29}{8}\times\frac{10}{23}=\frac{145}{92}$

c) $2\frac{3}{7}\div2.3=\frac{17}{7}\div\frac{23}{10}=\frac{17}{7}\times\frac{10}{23}=\frac{170}{161}$

8) a) $\frac{3}{5}+\frac{2}{7}(\frac{4}{5}+\frac{3}{2})=\frac{3}{5}+\frac{2}{7}(\frac{8}{10}+\frac{15}{10})$

$=\frac{3}{5}+\frac{2}{7}(\frac{23}{10})$

$=\frac{3}{5}+\frac{23}{35}$

$=\frac{21}{35}+\frac{23}{35}$

$=\frac{44}{35}$

b) $\frac{4}{3}\div(\frac{5}{2}-\frac{6}{7})=\frac{4}{3}\div(\frac{35}{14}-\frac{12}{14})=\frac{4}{3}\div\frac{23}{14}=\frac{4}{3}\times\frac{14}{23}=\frac{56}{69}$

c. $\frac{\frac{1}{6}\div(\frac{1}{3}+\frac{4}{5})}{\frac{5}{2}+\frac{1}{3}(\frac{2}{5}\div\frac{3}{5})}=\frac{\frac{1}{6}\div(\frac{5}{15}+\frac{12}{15})}{\frac{5}{2}+\frac{1}{3}(\frac{2}{5}\times\frac{5}{3})}=\frac{\frac{1}{6}\div(\frac{17}{15})}{\frac{5}{2}+\frac{1}{3}(\frac{2}{3})}=\frac{\frac{1}{6}\times\frac{15}{17}}{\frac{5}{2}+\frac{2}{9}}=\frac{\frac{15}{102}}{\frac{49}{18}}$

$=\frac{15}{102}\times\frac{18}{49}=\frac{90}{1966}$

9) $\frac{3}{4}$

10) I = p×R×T from this you can calculate the time as:

T = $\frac{I}{PR}=\frac{80}{500\times\frac{8}{100}}=\frac{80}{40}=2$

Therefore, the time is 2 years.

UNIT 2

Squares, square roots, cubes and cube roots

Total allotted period: 25 periods

Introduction

You give opportunities for the students to recall about

- multiplication of two similar factors,
- multiplication of three similar factors,
- area of a square and
- Volume of cuboids. Then students have understood they are familiar with the concept of squares, square roots, cubes and cube roots.
- Explain notation square and square roots and cubes and cube roots

Learning Outcomes: At the end of this unit, learners will be able to:

- Understand the notion square and square roots and cubes and cube roots
- Determine the square of numbers
- Determine the square roots of the perfect square numbers
- Extract the approximate square roots of numbers by using the numerical table and scientific calculator.
- Determine the cube of numbers
- Extract the cube roots of perfect cubes.
- Apply squares, square roots, cubes and cube roots in the real-life situation

2.1 Squares and Square roots

Period Allotted: ***11 periods***

Competencies

At the end of this sub-unit, students should be able to:

- Calculate the square of a number

Introduction

The square of a number is that number raised to the power 2.

Thus, if 'a' is a number, then the square of a is written as a^2 and is given by $a^2 = a \times a$. That is, the square of a number is obtained by multiplying it by itself.

If $a \times a = b$ i.e. $a^2 = b$, then we say that the square of number a is number b or the number b is the square of number a.

For example: 1) $3^2 = 3 \times 3 = 9$, so we say that the square of 3 is 9;

2) $(-4)^2 = -4 \times -4 = 16$, so we say that the square of -4 is 16;

3) $(3/5)^2 = (3/5) \times (3/5) = 9/25$ so we say that the square of (3/5) is 9/25;

2.1.1 Square of a Rational Number

Competency: At the end of this sub- unit students should:

- ❖ Calculate the square of a number

Introduction

Here students have background idea on multiplication of rational numbers. Then you can start this lesson by asking such questions

1. Compute the following
   a) 5×2
   b) $\frac{3}{4} \times \frac{5}{2}$

2. Compute the products for each of the following
   a) 3×3
   b) 0.5×0.5

   You can use examples similar to the examples given in the text book. Remind the students that most of the values obtained from the table are approximate values. Guide students to use the table because it is time saving.

   The purpose of Activity 2.1 is introducing the concept of squaring just by counting the number of unit squares contained in the square region. So you should give much emphasis on the ways or forms of expressing the results that are supposed to be filled in a –d square. Answer to activity 2.1 are as follow as.

   Answer for Activity 2.1

   1. a. 1 b. 9 c. 16 d. 25
   2. i. 20 ii. 15 $\frac{175}{20} = 8\frac{15}{20}$
   3. a.36 b. 100 c. 0.25

   Answer for group work 2.1

   a. 1+3+5+7+9+11+13+15+17+19+21+23+25+27+29 = 225
      and it is written as $15 \times 15 = 15^2$
      then, the sum of the first fifteen odd numbers is 15^2
   b. The sum of the first n odd numbers is n^2
   c. We conclude that, the sum of the first n odd numbers isn^2.

Encourage your students to square fractions. Whether it is positive or negative by using the concepts of multiplication of fraction,. You should also guide the students to square decimals

after changing them to fraction and express the result in decimal form. The possible answer to Exercise 2.1 is as follows

Answer for exercise 2.1

1. a. true b. False c. False d. false e. true
2. a. 36 b. 400 c. $\frac{1}{16}$ d. $\frac{25}{16}$

   e. 0.0009 f. 20.25 g. $\frac{144}{16} = 9$
3. 4 ,9 , 16 , 36 , 0.04 , $\frac{1}{4}$, $\frac{1}{121}$, 0.01 , 225
4. a. is a square number

   b. not a square number

   c. is a square number
5. 196 , 400 , 324
6. 121 , 225 , 529
7. i, 75 ii, 149 iii, 183 iv, 209 v, 281 vi, 435

As you have seen the pattern, the difference of the square of two consecutive rational number is the sum of the numbers.

8. 64 = 1+3+5+7+9+11+13+15

Assessment

Dear teacher you are strongly you are strongly advised to give tutorials to the students by dividing them into three groups such as slow learners, bright learners and fast learners.

Finally ask your students orally to read and give meanings to squares of numbers. In addition to this for slow learners and fast learners, you can also give the following additional Exercise problems.

For slow learner students

1. Can you define a squaring a number?
2. Find the product

   a) $3^2 \times 5^2$ b) $3^2 \times 5^2 \times 3^2$ c) $2^2 \times 6^2 \times 4^2$

For fast learner students

3. If $x = 3^2 + 6^2$, then What is the value of x?
4. What is the area of a square whose sides is:

   a) 6cm b) 8cm c) 25cm d) 100cm

2.1.2 Use of table values and scientific calculator to find squares of rational numbers

Competencies

At the end of this sub-unit, students should:

- Calculate the square of a number

In the previous lesson students had learnt about scientific notation, scientific notation is one ways to find square roots of rational numbers.

To find the square of rational number when it is written in the form of a decimal is very tedious and time consuming work so, To avoid this tedious and time consuming work you have to consult students how they can find the square root of a rational numbers from numerical a table value and student can use scientific calculators.

You can start the lesson recalling

1. Write the following numbers in scientific form.
   a. 26.4 b. 226 c. 0.226

Then students remembered that how they can do the following activity 2.2 to find the square roots of rational numbers by using numerical table value.

Answer for activity 2.2

1. A number is said to be in scientific notation (or standard form), if it is written as a product of the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.
2. a. 4.5×10^2 c. 8.43×10^1
   b. 4.5×10^{-3} d. 2.56×10^{-1} e. 5×10^{-2}
3. a. 2.280 b. 10.56 c. 25.60
   d. 11.56 e. 27.98 f. 58.37

This problem given in exercise 2.3 are calculated the square of rational numbers and the square roots of rational numbers. So encourage your students to do all of them

Answer for exercise 2.3

1. a. false b. false c. false
2. *if* $(3.67)^2 = 13.47$
   a. $(36.7)^2 = (3.67 \times 10)^2 = (3.67)^2 \times (10)^2 = 13.47 \times 100 = 1347$
   b. $(367)^2 = (3.67 \times 100)^2 = (3.67)^2 \times (100)^2 = 13.47 \times 10000 = 134700$
   c. $(0.367)^2 = (3.67 \times \frac{1}{10})^2 = (3.67)^2 \times (\frac{1}{10})^2 = 13.47 \times \frac{1}{100} = 0.1347$
3. *If* $(8.435)^2 = x$, *then*
   a. $(84.35)^2 = (8.435 \times 10)^2 = (8.435)^2 \times (10)^2 = x \times 100 = 100x$
   b. $(0.8435)^2 = (8.435 \times \frac{1}{10})^2 = (8.435)^2 \times (\frac{1}{10})^2 = x \times \frac{1}{100} = \frac{x}{100}$

4. a. $8.95 \approx 9$, then $(8.95)^2 \approx 9^2 = 81$

   b. 80.01 c. 80.0125

For slow learner students

1. Find the product of the following

   a. $(\frac{5}{4})^2$

   b. $\frac{2}{3} \times \frac{2}{3}$

   c. 0.02×0.02

2. Find the area of a square plot of land whose each side measures $\frac{13}{2}$ meters.

For fast learners

1. Show that the differences between any two consecutive square numbers are an odd number.
2. Show that the difference between the 7^{th} square number and the 4^{th} square number is a multiple of 3

2.1.3 Square Roots of a rational number

Competencies

At the end of this sub-unit, students should:

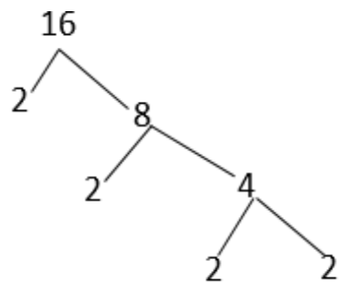
- ❖ Calculate the square root of perfect squares.

Introduction

In the previous sub-topics students are introduced to the concept of squaring and how to find the square of rational numbers. In this sub topics they will be introduce the reverse process of rational number which is square roots of rational numbers. The new concept is square roots of perfect square. Here you strongly begin the square roots of perfect squares, otherwise students confused to find the square roots of rational number which is not perfect. For instance $\sqrt{8}$ this is not possible in this grade level. You encourage the students to do the following activities

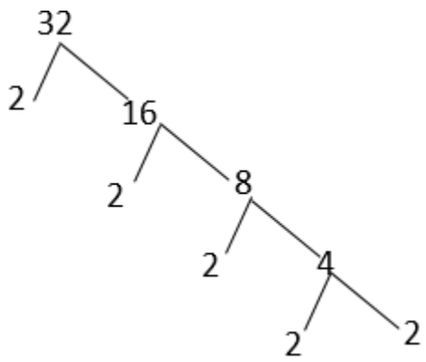
Answer for activity 2.3

1. a.



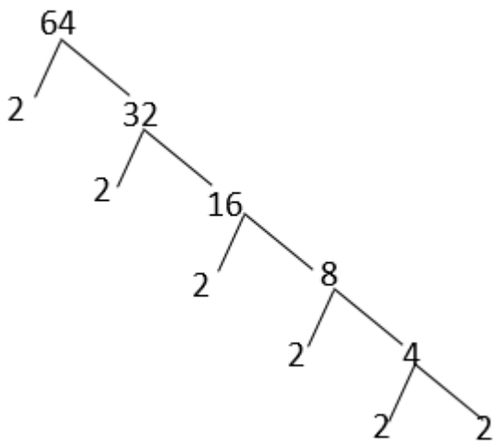
$16=2\times2\times\ 2\times\ 2 = 2^4$

b.



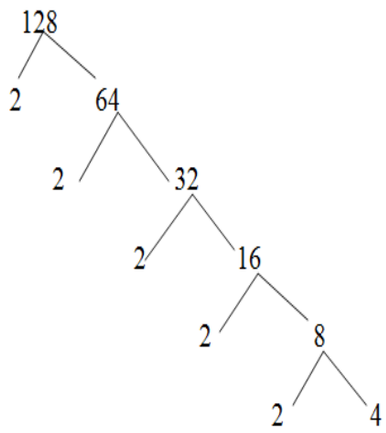
$32=2\times2\times\ 2\times\ 2\times\ 2=2^5$

c.



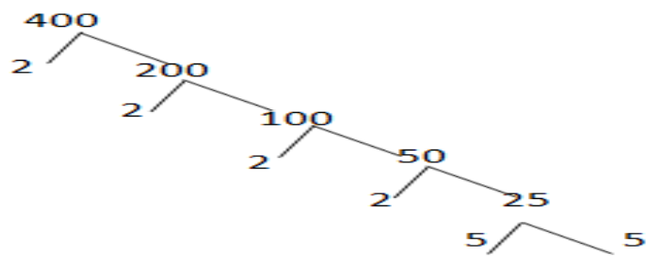
$64 = 2\ 2\times\ 2\times\ 2\ 2\times\ 2 =2^6$

d)



$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

e.



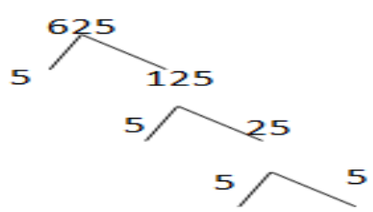
$$400 = 2\times2\times 2\times 2\times 5\times 5 = 2^4 \times 5^2$$

Additionally, in exercise 2.4 you can encourage your students how they can find the square roots of rational numbers by using prime factorization method

Answer for exercise 2.4

1. a. 0 b. 13 c. 24 d. 0.5 e. 0.04
2. 12 length

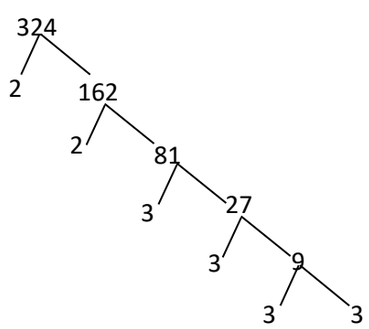
3. a.



$$625 = 5\times5\times5\times5 = (5\times5)\times (5\times5) = 25\times25$$

$$\text{Therefore, } \sqrt{625} = 25$$

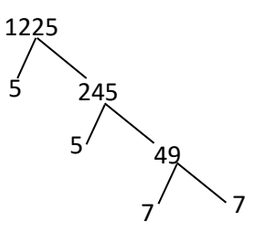
b.



$324 = 2\times2\times 3\times 3\times 3\times 3 = (2\times3\times 3)\times (2\times3\times 3) = 18\times18$

Therefore, $\sqrt{324} = 18$

c.



$1225 = 5\times5\times 7\times 7 = (5\times7)\times(5\times7) = 35\times35$ is

Therefore, $\sqrt{1225} = 35$

Assessment

Always remember to consider the minimum learning competencies that are expected of students. As part of your assessment technique, it mainly indicates that your students. So ask them to read the required root or number from the square table. Give questions and ask students to find square root of non-negative numbers and compare their result by computing, by using table of square and by using scientific calculator. Depending on their level of understanding you may also give additional exercise of the following type:

For slow learner students

Answer the following question

1. Can you define a square root?
2. Find the square root of 324
3. Simplify

   a. $\sqrt{8+17}$ b. $\sqrt{125-44}$ c. $\sqrt{49\times 4}$

For fast learner students

4. Simplify each of the following

a. $\sqrt{8}$ b. $\sqrt{12}$ c. $\sqrt{64+132}$ d. $\sqrt{\frac{32}{2}}$ e $\sqrt{27}$

5. Find the length of sides of a square whose area is $(144cm)^2$.

2.1.4 Use of table values and scientific calculator to find square roots of rational numbers

Competencies

At the end of this sub-unit, students should:

- ❖ Calculate the square root of perfect squares.

Introduction

In previous, students discussed about the square root of perfect square. But, in this sub-topic the ways how we can find the square roots of decimal numbers is presented by use of table value and scientific calculator to find square roots of rational numbers. You will be motivate your students by explaining the importance of square table and make sure that they fully understand how to read and find the square roots of a numbers from the end of the text.

Answer for activity 2.4

1. a. 3.63 b. 3.34 c. 3.57 d. 9.90 e. 7.54

Dear teacher, if the radicand is not found in the body of the table, you can approximate to the nearest square roots of a numbers. Motivate your students to do exercise 2.5

Answer for exercise 2.5

1. a. 1.62 b. 2.08 c. 3.96 d. 9.93 e. 9.75
2. if $(4.63)^2 = 21.44$
   a. $\sqrt{21.44} = 4.63$
   b. $\sqrt{2144} = \sqrt{21.44 \times 100} = \sqrt{21.44} \times \sqrt{100} = 4.63 \times 10 = 46.3$
   c. $\sqrt{0.2144} = \sqrt{21.44 \times \frac{1}{100}} = \sqrt{21.44} \times \sqrt{\frac{1}{100}} = 4.63 \times \frac{1}{10} = 0.463$

Assessment

Dear teacher you have to motivate your students using square table to find the square roots of decimal numbers and initiate your students using calculator.

Finally based on the performances of your students,

- Asking oral questions.
- Giving your own assignment
- Giving quiz, Test and project work

Then give additional exercise of the following type:

For slow learner students

1. Find the square root of each of the following numbers using numerical table value.
   a. 18.82 b. 3.112 c. 112.0

For fast learner students

Find the square root of each of the following numbers

a. 7.762 b. 77.62 c. 776.2 d. 7762 e. 0.7762

2.2 Cubes and cube roots

Allotted period: ***period -11***

2.2.1 Cube of rational number

Competency: At the end of this sub-topic students should:

- ❖ Calculate the cube of a number

Introduction

In this sub-topics students should be calculate the cubes of a rational numbers and you can start the lesson by guide your students how to find the volume of cuboids and guide your students to realize that when a number is decomposed in to a product of three equal factors.

You may start the lesson by giving the following question

1. Find the cube of each of the following numbers

   a. 3 b. $\frac{3}{4}$ c. -3

2. Find numbers whose cube

   a. 8 b. 64

Answer for activity 2.5

1. a. 8 b. -64 c. $-\frac{1}{64}$ d. 0.125 e. 0

2. 8 , 64 , 729

Answer for Group work 2.2

Please guide the students to discuss the pattern that shown in group work 2.2 on students' text book and ask the students what you observe from the pattern?

The sum of the next eight odd numbers started from the last pattern is $57+59+61+63+65+67+69+71 = 8^3$

The sum of the first ten consecutive odd numbers is 100.

The sum of the first 21 consecutive odd numbers is 441.

Dear teacher please given proper feedback for each students and correct it exercise 2.6

Answer for exercise 2.6

1. a. false b. false c. true
   d. false e. true f. true

2. a. 64 b. 0.125 c. $-\frac{1}{8}$ d. $-\frac{125}{64}$

3. a. -27 b. 125 c. 0 d. 27

4. a. 64, 125 and 216 are perfect cubes

5. 1, 8, 27 and 64 are the consecutive perfect cube numbers whose sum is 100.

1, 8, 27, 64, 12 and 216 are the consecutive perfect cube numbers whose sum is 441.

Assessment

Dear teacher asses your students' based on the following sub-topics:

- ❖ Give different exercise (reading tables of cubes)
- ❖ Ask students orally to answer simple cubes of a numbers
- ❖ Ask your students to find the cube of a number and compare their result using computing, using table of square and scientific calculator
- ❖ Finally based on the performances of your students.

- Asking oral questions.
- Giving your own assignment
- Giving quiz, Test and project work

Then give additional exercise of the following type:

For slow learner students

1. Define perfect cube a number
2. Which one of the following number both perfect cube and perfect square

   a. 8 b. 64 c. 9 d. 40

For fast learner students

3. Identify whether each of the following numbers are perfect cubes

   a. 216 b. 729 c. 625 d. 3375

2.2.2. Cube Root of a rational number

Competency: At the end of this sub-topic students should:

- ❖ Calculate cube roots of a rational number

Introduction

In previous, students discussed about the cube of perfect square. But, in this sub-topic the ways how we can find the cube roots of decimal numbers is presented by use of table value and scientific calculator to find cube roots of rational numbers. You will motivate your students by explaining the importance of square table and make sure that they fully understand how to read and find the cube roots of a numbers from numerical table.

Answer for activity 2.6

1. students may be defining cube roots of a rational number in different ways, so you observe what students said that.
2. a. 8 b. 9 c. 7 d. 4 e. 11

Motivate your students by telling them the table of cube to simplify the numbers. Additional you should also tell them the given numbers in the numerical table to approximate value easily. The possible answer for exercise 2.7 as follows

Answer for exercise 2.7

1. cube root of 4
2. a. $10^{\frac{1}{3}}$ b. $(0.23)^{\frac{1}{3}}$
3. 8, 64, 216, 729 are perfect cubes.
4. a. 6 b. -7 c. 10 d. -12 e. 0

Assessment

- ❖ Give exercise 2.7 as a class work and home work which are given in the text and to take remedial measures based on their feedback.
- ❖ Assist your students to use internet and explain the relation between cubing and extracting cube root from the cube of a number.

For low learner students

1. Define cube Roots
2. Evaluate each of the following
   a. $\sqrt[3]{8}$ b. $\sqrt[3]{27}$ c. $\sqrt[3]{64}$
3. Find the sides of a cube whose volume is $343cm^3$.

For fast learner students

4. Evaluate each of the following
   a. $\sqrt[3]{216}$ b. $\sqrt[3]{3375}$ c. $\sqrt[3]{54}$ d. $\sqrt[3]{16}$

2.3 Applications on squares, square roots, and cubes and cube roots

Period Allotted – 3periods

Competency: At the end of this sub-topic students should:

- ❖ Solve real-life problems

Introduction

In this sub-unit will be solve real-life application on square, square roots, cubes and cube roots applying in carpentry, architecting, engineering, and etc. Guide your students how square and cubes related with real- life applications, motivate your students to lists some examples that related with their day to day activities.

Answer for activity 2.7

1. a. $As = 2 \times (l \times w + l \times h + w \times h)$

   $= 2 \times (6 \times 6 + 6 \times 6 + 6 \times 6) = 2 \times (36 + 36 + 36) = 2 \times (108) = 216$

   Therefore, the surface Area of a cube is $216cm^3$.

   b. $v = l \times w \times h = (6 \times 6 \times 6) = 216$

   therefore, the volume of a cube is $216cm^3$

   c. here the surface area and volume of a cube is the same which is $216cm^3$.

   2. $A = l \times l = (5cm)^2 = 25cm^2$.

Answer for exercise 2.8

Solution:

1. Let x be the number of rows and y be the number of students in each row but the number of rows and number of students in each row is equal.

$1225 = x^2$

$x = \sqrt{1225} = 35$

2. $(s+1)(s+3) = s^2 + 19$

$s^2 + 4s + 3 = s^2 + 19$

$s = 4$

3. $A = s^2$

$s = \sqrt{64} = 4$

Assessment

Dear teacher you will list some applications that related with real-life of students to solve those problem by the easiest ways. Use cube root to solve for the dimensions of a three-dimensional object of a certain volume.

Dear teacher you may use group discussions, class work, homework, and quiz to make students to present it in the classroom.

Answer for Review Exercise

1. a. false b. true c. true d. false e. false f. true
2. *a.* $x = 0.0009$

   b. $x = 4$

   c. $x = \frac{1}{16}$

   d. $x = \frac{25}{16}$
3. *625, 900 and 2025 are perfect squares*
4. n^2
5. *a. 64 b. 64.64 c. 64.6416*
6. *a.* $x = 8$ *or* $x = -8$

   b. $x = 9$ *or* $x = -9$

   c. $x = 0.2$ *or* $x = -0.2$
7. *You can use prime factorization method, then*

   a. $3^2 \times 5^2$

   b. 5^4

   c. $2^2 \times 5^4$
8. *a. 4.43*

b. 3.25

c. 5.48

d. 8.04

9. a. 5520 *b. 552000* *c. 0.552*

10.a. 3.42 *b. 34.2* *c. 0.342*

11. 27 and 64 are perfect cubes.

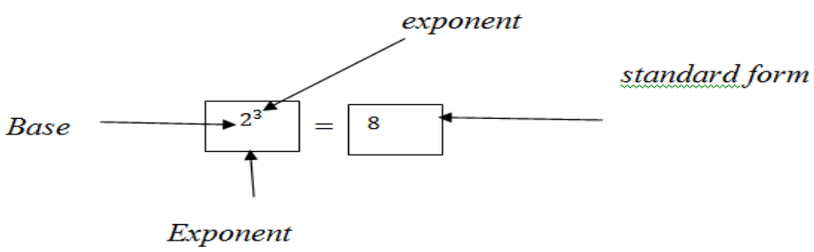
12. a. $x = 8$

b. $x = 0.000027$

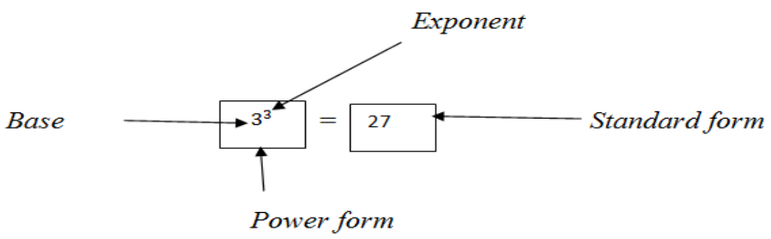
c. $x = -8000$

d. $x = \frac{1}{64}$

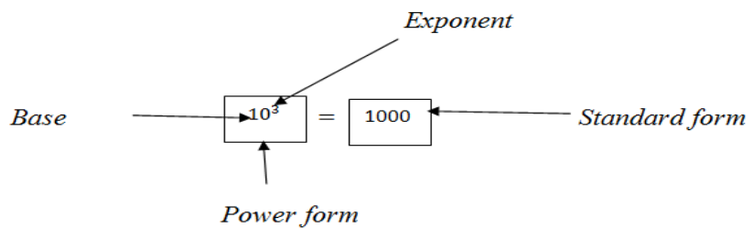
13.a.



b.



c



14.a. 30^2 *b.* 18^2 *c.* 6^2

15.31+33+35+37+39+41= 216

16.a. $As = AL + 2A_B$

$As = Ph + 2(l \times w)$

$$As = (4cm + 4cm + 4cm + 4cm)4cm + 2(4cm \times 4cm)$$

$As = 64cm^2 + 32cm^2 = 96cm^2$

b. $v = l \times w \times h = 4cm \times 4cm \times 4cm = 64cm^3$

c. *Therefore, surface area of cube is greater than the volume of the cube.*

17. $A = l^2 = (10cm)^2 = 100cm^2$

UNIT 3

LINEAR EQUATIONS AND INEQUALITIES

Total Allotted Period: 21 Periods

LEARNING OUTCOMES: At the end of this unit, learners will be able to:

- Graph linear equations of the type $y = mx + c$.
- Solve linear inequalities.
- Solve applications of linear inequalities.

INTRODUCTION:

The main purpose of this unit is to extend the knowledge of the students about the Cartesian coordinate system and graphs of a linear equations. Furthermore, the students are required to be familiar with the x- axis and the y- axis which divide the coordinate plane into four quadrants namely I, II, III, and IV quadrant. The students should also be able to plot points on the coordinate plane representing given ordered pairs and conversely to read the coordinates of the given points on the coordinate plane and draw lines of the form

$y = mx + c, m \in \mathbb{Q}, m \neq 0$

In addition to this, the unit is formed on solving linear inequalities by means of a systematic discussion of rules for transforming inequalities. It is presented in such a way that students will develop skills in solving real life word problems systematically and independently.

Suggested Teaching Aids in Unit 3

- ✓ You can present different flow chart and graphs to demonstrate linear equations.
- ✓ You can also encourage students to prepare different representative graphs of linear equations by themselves.
- ✓ You can present chart showing the four quadrant of the coordinate plane.

Assessment

Generally, it is not necessary to assess all students every day. What is important is to keep track of their learning so that students who are lagging behind are identified and given extra help. And students who are succeeding and moving along quickly are given more challenges to keep them being simulated and learning.

3.1. Revision of Cartesian coordinate system

Period Allotted*: 1 period*

Competency: At the end of this section, students should:

- Describe the Cartesian coordinate system.

Introduction: This sub- unit mainly deals with Cartesian coordinate system. The concept we discussed in the sub-unit enables students to plot points on the xy-coordinate plane and read

coordinates of the given points on the coordinate plane. Students are expected to have background on Cartesian coordinate plane. So, it is advisable to encourage students by giving time to discuss Activity 3.1. The main purpose of Activity 3.1 is to let you know and identify students who read and prepare themselves ahead. For this activity, especially question number one, you may use asking- answering methods. Don't give much emphasis on the correct answers but appreciate their bravery. The possible answers for activity 3.1

Answer for Activity 3.1

1.a. Cartesian coordinate plane is a system in which the location of a point is given by coordinates that represent its distance from perpendicular lines.

b. Ordered pair is a pair of numbers that can be written in the form of (a, b).

c. A quadrant is the region enclosed by the intersection of the x-axis and the y-axis. On the Cartesian plane when the two axes, x-axis and y-axis, intersect with each other at 90^{o}, there are four regions formed around it. And their regions are called quadrants.

2.

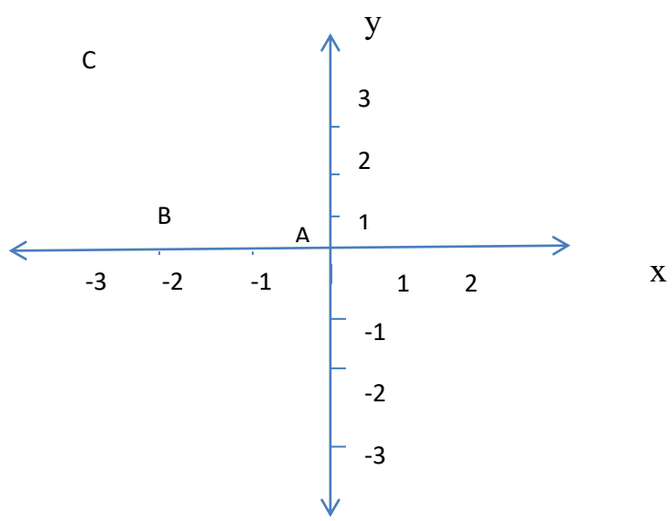


Figure 3.1

3. a) quadrant IV b) quadrant II c) quadrant III

4 a) quadrant IV b) quadrant III d) quadrant II

5.a. If, x = -1, then y= 3(-1)-1= -4

If, x = 0, then y = 3(0)-1 = -1

If, x = 1, then y = 3(1) – 1 = 2

b. The ordered pairs are (-1, -4), (0, -1) and (1,2).

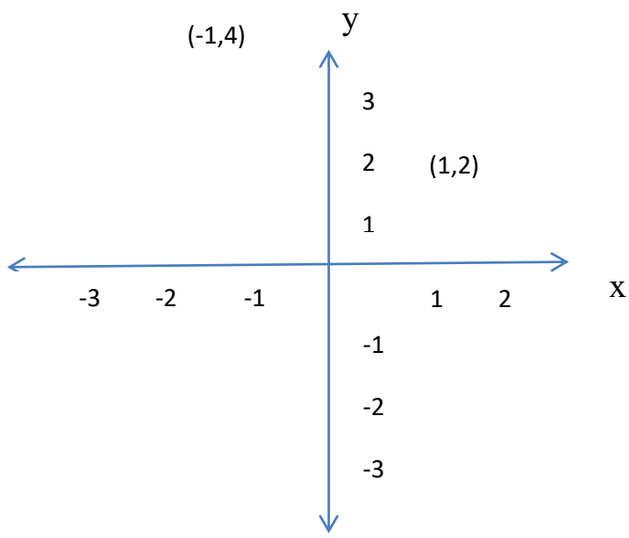


Figure 3.2

Answer for exercise 3.1

1. a. quadrant IV
   b. quadrant II
   c. quadrant III and IV
   d. quadrant III
   e. quadrant I
   f. quadrant I and IV

2. . a. (-5,2)
   b. (-10,0)

You can give additional exercise of your own other than the ones given in the student text book.

3.2. Graphs of linear Equations

Period allotted: 8 periods

Competency: At the end of this unit, students should:

- Draw linear equations like $y = mx + c$ in a Cartesian coordinate plane.

Introduction:

The students are already familiar with graphs of horizontal and vertical lines, and lines that pass through the origin. Start the lesson by drawing table of values for x and y in which y – is constant and following this let students plot these points and realize that the points lie on a horizontal line. You may take example like:

x	-3	-2	-1	0	1	2	3
y	4	4	4	4	4	4	4

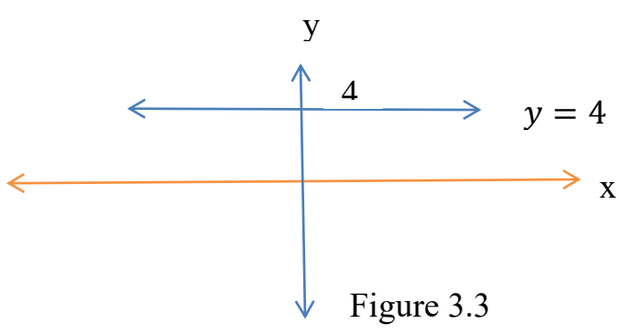


Figure 3.3

Assist students to come to the conclusion that $y = a$ where $a \in \mathbb{Q}$ is an equation of a horizontal line (or a line parallel to the x-axis). Do the same thing as suggested above, for vertical line, and assist students to come to the conclusion that $x = b$ where $b \in \mathbb{Q}$ is an equation of a vertical line (or a line parallel to the y-axis). After giving a brief summary of graphs of vertical and horizontal lines, give activity 3.2 as a group work.

Answer to activity 3.2

1.

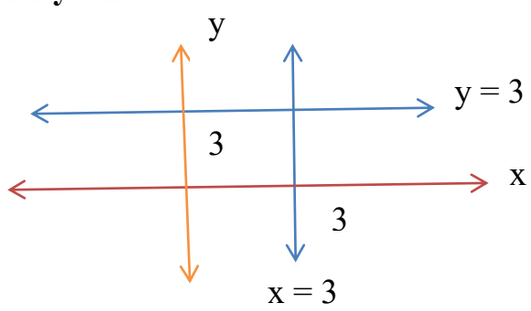


Figure 3.4

2. a. True b. True

c. False d. False

3. a. y = 0 b. x = 0

4.

x	-3	0	2
$5x - 2$	-17	-2	8

5. $3x + 5y = 4$

$5y = -3x + 4$

$y = -\frac{3}{5}x + 4$

6. a) on the left side of the y-axis

b) y- axis

c) on the right side of the y-axis

7. y –coordinate constant, x – coordinate varies

Answers to activity 3.3

1a) An equation that can be written in the form $ax + b = 0, a \neq 0$,is called a linear equation in one variable.

b) An equation of the form $ax + by = c, a \neq 0, b \neq 0$ is a linear equation in two variables.

2) $3x + 1 = 4 - x$ is a linear equation in one variable.

3i) a. one solution b. no solution c. infinite solutions

ii) a linear equation in one variable has exactly one solution, $x = -\frac{b}{a}$

4) b and d

5a) only (0, 0) and (2, 8) satisfy the equation.

b) (0, 0) and (2, 8) are on the line.

c) Every point on the line are solutions of the equation.

6) A line $y = mx, m \neq 0$($y = -4x, y = 3x$) and $x = 0, y = 0$ are lines that passes through the origin.

Answers to Exercise 3.2

1a)

x	-1	0	1	2
$y = -5x$	5	0	-5	-10
(x, y)	(-1, 5)	(0, 0)	(1, -5)	(2, -10)

2)

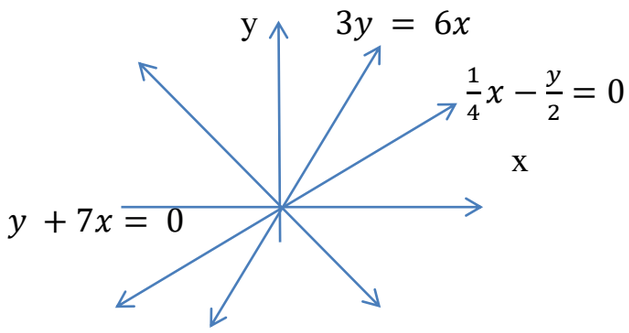


Figure 3.5

3) ($a, 3$) is on the line $12x - 2y = 0$means the point satisfies the equation.

Therefore, $12\,a - 2(3) = 0$

$12a = 6$

$a = \frac{1}{2}$

4a) $y = -3$ b) $x = 4$ c) yes

Answers to Exercise 3.3

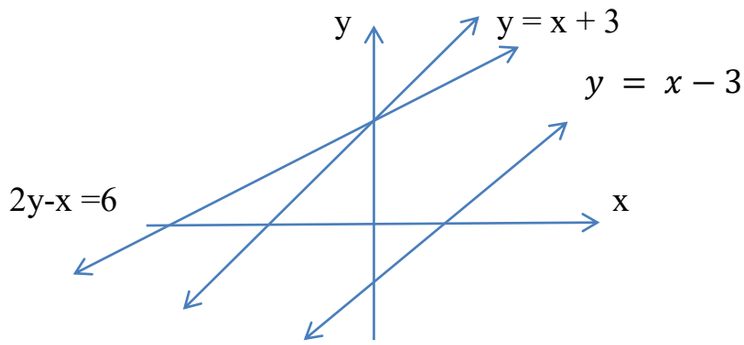


Figure 3.6

3.3. Solving Linear inequalities

Period Allotted: 8 periods

Competency: At the end of this section, students should:

- ❖ Solve linear inequalities.

Introduction:

This sub-unit mainly deals with solution of linear inequalities. The concept discussed in this sub- unit enables students to find solution of linear inequalities involving brackets, fractions and solve problems using linear inequalities.

You can start this lesson by reviewing solutions of equations given in activity 3.4 question number1. This activity is mainly to remind the students how to solve linear equations in one variable. With active participation of students, you need to assist them to find the solution of a given linear equations. After you make sure that students have grasped the techniques of solving linear equations, extend it for solving linear inequalities by guiding students to solve linear inequalities given in activity 3.4 question number 3.

Answers to Activity 3.4

1a) x = 18 b) x = 4 c) x =5

d) $(3x - 1) - 4(2x + 1) = 4(4x - 3)$

$3x - 1 - 8x - 4 = 16x - 12$

$-5x - 5 = 16x - 12$

$7 = 21x$

$x = \frac{1}{3}$

2) a and b.

3a) $10x < 23$

$x < \frac{23}{10} = 2.3$

In W: $x = 0, 1, 2$

In $\mathbb{Z}: x = \ldots -3, -2, -1, 0, 1, 2$

In $\mathbb{Q}: x < 2.3$

b) In W: no solution

In $\mathbb{Z}$: … -5, -4, -3

In $\mathbb{Q}: x < -2.5$

c) In W: 8, 9 , 10,…..

In $\mathbb{Z}$: 8, 9, 10, ….

In $\mathbb{Q}: x \geq 8$

d) In W: 0, 1, 2, 3, 4

In $\mathbb{Z}$: …, -2, -1, 0, 1, 2, 3, 4

In $\mathbb{Q}: x \leq 4$

Answers to Exercise 3.4

1a) ≤ b) ≥ c) <

2a) equivalent b) not equivalent

c) equivalent d) equivalent

Answers to exercise 3.5

1a) $2y - 3 < \frac{1}{2}(7 - y)$

$2y - 3 + 3 < \frac{7}{2} - \frac{y}{2} + 3$

$2y < \frac{13}{2} - \frac{y}{2}$

$2y + \frac{y}{2} < \frac{13}{2} - \frac{y}{2} + \frac{y}{2}$

$\frac{5}{2}y < \frac{13}{2}$

$\frac{2}{5} \times \frac{5}{2}y < \frac{13}{2} \times \frac{2}{5}$

$y < \frac{13}{5}$

In a similar way the must shows the steps for the students of the remaining questions.

b) $x \geq -\frac{14}{5}$ c) $x \leq -\frac{4}{3}$

2a) no values of x satisfy the inequality.

b) all rational numbers satisfy the inequality.

3.4. Applications of Linear Equations and Inequalities

Period Allotted: 4 periods

Competencies: At the end of this section, students should:

- Apply linear equations and inequalities in real life situation.

❖ Solve linear equations and inequalities real- life problems.

Introduction:

The modern way of teaching mathematics is to relate or apply it to the real world situation. When you do this it helps you to motivate your students so that they appreciate and like the subject. So, you can present simple and more interesting problems from day- to –day activities that the students are familiar. Your examples and problems should be solved by using directly what you teach in the class. Here are possible answers to exercise 3.6 and 3.7 which are the applications of linear equations and linear inequalities in real life situation.

Answers to exercise 3.6

1a) $7x - 5 = 0$ b) $\frac{x}{9} = x - 2$ c) $2x + 4 = 3x - 7$

2) $F = G\frac{mM}{r^2}$

$Fr^2 = GmM$

$M = \frac{Fr^2}{Gm}$

3) B + G = 42 and G = 1.1 B

So, B + 1.1B = 42

2.1 B = 42

B = $\frac{42}{2.1} = 20$ and G = 42 – B = 42 – 20 = 22

4) let x be the age of the mother and y be the age of her daughter.

Then, x + y = 68 and x = y + 22

y + 22 + y = 68

2y = 46

y = 23 and x = y + 22 = 45

Answers to exercise 3.7

1a) $3x - 4 > 20$ b) $\frac{x}{7} < 10$

c) $h \leq 10$ d) $x + (x + 1) < 3x$

2. Let x be the number. Then,

$5x - 2x < 12$

$3x < 12$

$x < 4$

Therefore, the possible values of x are 1, 2, and 3

3.Let x be the total km covered by the trip. Then,

$50 + 10x \leq 1350$

$10x \leq 1300$

$x \leq 130$ km

Assessment

After completing this unit, you can use any of the following techniques for assessing students learning class activities, group discussion, assignments and quiz or test.

Dear teacher, please assess your students' based on the following concepts.

- ✓ Give different exercise problems to describe Cartesian coordinate system
- ✓ Give different exercises to solve linear equations
- ✓ Give different exercise problems to draw graphs of linear equations
- ✓ Give different exercise problems to solve linear inequalities
- ✓ Give different exercise problems to solve real life problem in application of linear equations and inequalities.

For slow learners

1) Draw the graphs of the following linear equations on the same coordinate plane.
   a) $y = \frac{2}{3}x + 6$
   b) $y = \frac{3}{4}x + \frac{7}{6}x + 2$
2) Take the following numbers: -2, -1, 0, 2, and 3 and determine which numbers satisfy the inequality.
   a) $3 - 2x \leq \frac{1}{2}$
   b) $\frac{1}{x} \leq \frac{1}{2}$
   c) $2x - 1 \geq x$

For fast learners

1) Draw the graph of the following linear equations on the same coordinate system
   a) $y = \frac{7}{3}x - \frac{4}{10}x + 8$
   b) $3x + \frac{12}{5}x + 2y = 0$
2) Solve the inequality and sketch the solution set.

   $4 \leq 3x - 2 < 13$

Solution:

The solution set consists of all values of x that satisfy both the inequalities $4 \leq 3x - 2$ and $3x - 2 < 13$.

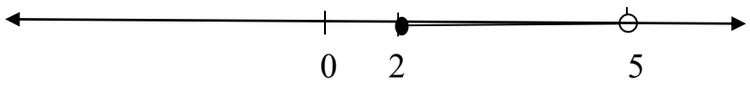
$4 \leq 3x - 2 < 13$

$4 + 2 \leq 3x - 2 + 2 < 13 + 2$

$6 \leq 3x < 15$

$\frac{6}{3} \leq \frac{3x}{3} < \frac{15}{3}$

$2 \leq x < 5$



3) Take the following numbers: $-2, -1, 0, \frac{1}{2}, 2, 3$ and determine which numbers satisfy the following inequalities

a) $1 < 2x - 4 \leq 7$

b) $-2 \leq 3 - x < 2$

c) $-\frac{1}{2} \leq \frac{4-3x}{5} \leq \frac{1}{4}$

d) $\frac{1}{3}x + 2 < \frac{1}{6}x - 1$

4) The instructions on a bottle of medicine indicate that the bottle should be stored at a temperature between 5^0c and 30^0c. what range of temperature this correspond to on the Fahrenheit scale?

Solution:

The relationship between degrees Celsius (C) and degrees Fahrenheit (F) is given by the equation: $C = \frac{5}{9}(F-32)$

Since $5 < C < 30$

$$5 < \frac{5}{9}(F-32) < 30$$

$$\frac{9}{5} \times 5 < (F-32) < \frac{9}{5} \times 30$$

$$9 < F-32 < 54$$

$$9 + 32 < F < 54 + 32$$

$$41 < F < 86$$

Therefore, the medicine should be stored at a temperature between 41^0F and 86^0F.

Answers to Review Exercise

1a) False b) True c) False d) True e) True

2)

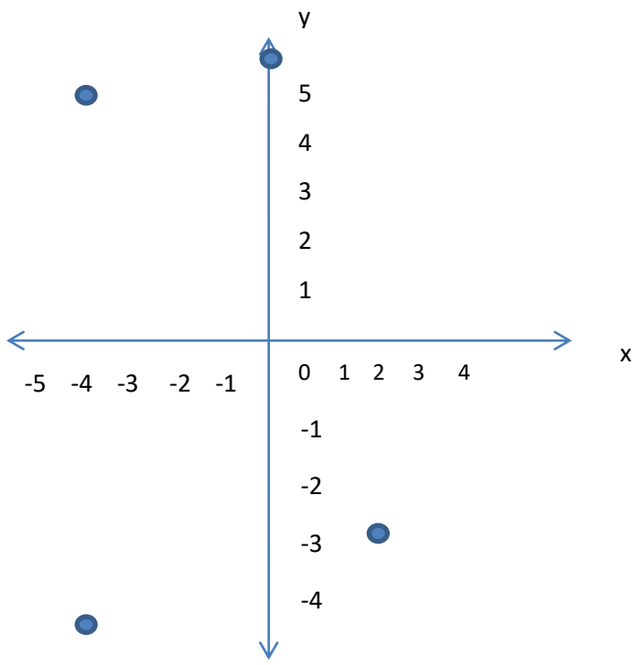


Figure 3.7

3) a) D(0,4) E(3,-1) F(2,0) b) point A

c) y coordinate d) point C III quadrant and pion E IV quadrant

4) a) ≤ b) > c) >

5) a) (0, 0) – not on the line

b) (1, 0) – on the line

c) (-1, -1) – on the line

d) (2, 1) – not on the line

6)

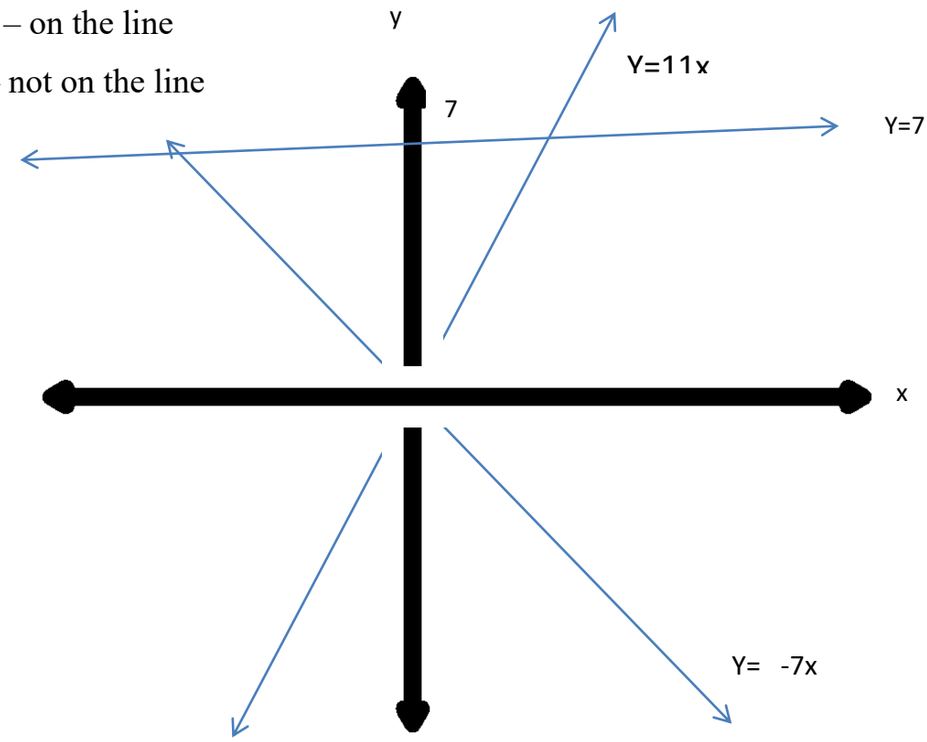


Figure3.9

7) Given the equation $ax - by = 6$, substitute the values of x and y to get two

equations in terms of a and b

where $x = 3$,and $y = 1$, $3a - b = 6$ and

where $x = 0$,and $y = 2$, $-2b = 6$ from this we get $b = -3$.

Now substitute the values of $b = -3$ in the equation

$$3a - b = 6$$

$$3a - 3 = 6$$

$$a = \frac{6-3}{3}$$

$$a = 1$$

Therefore, the values of a and a, are $a = 1$, $b = -3$

8)

x	-1	0	1
$y = x + 1$	0	1	2
$y = 1 - x$	2	1	0

b) (-1,0),(-1,2),(0,1),(1,2),(1,0)

c) (0, 1) is an intersection point

9) a) $3x + 11 \leq 6x + 8$

$3x - 6x \leq 8 - 11$ ----------collect like terms

$-3x \leq -3$

$\frac{-3x}{-3} \geq \frac{-3}{-3}$ ----------divide both sides by -3

$x \geq 1$

b) $6 - 2x > x + 9$

$-2x - x > 9 - 6$

$-3x > 3$

$x < -1$

c) $4 - 3x \leq -\frac{1}{2}(2 + 8x)$

$4 - 3x \leq -1 - 4x$

$-3x + 4x \leq -1 - 4$

$x \leq -5$

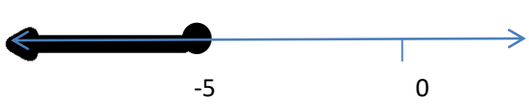


Figure 3.10

10) a) $\frac{x}{7} \geq \frac{3}{14}x - \frac{1}{7}$

$2x \geq 3x - 2$

$2x - 3x \geq -2$

$x \leq 2$

b) $2\left(\frac{1}{2} - x\right) < 3\left(1 + \frac{1}{2}x\right) + 5$

$2 - 2x < 3 + \frac{3}{2}x + 5$

$-2x - \frac{3}{2}x < 3 + 5 - 2$

$-\frac{7}{2}x < 6$

$x > -\frac{12}{7}$

c) $x \leq -6$

d) $y \leq \frac{7}{4}$ e) no solution f) All rational numbers

11. a) 1, 2, 3

b) No whole numbers satisfy the inequality.

c) All rational numbers satisfy the inequality.

12) a) $F - 1 > 3(S - 1)$

b) $\frac{3}{4}x > 12$

c) $x \geq 89$

13) $x = 1.1m, \quad y = 1.4m$

14) 5 years

15) x = 13

16) G = 0, 1 and S = 3, 4

UNIT 4

SIMILARITY FIGURES

Total Period Allotted: 14 periods

Introduction:

The unit focussed on similarity of plane figures. Similarity focuses on shape rather than size. In order to develop this new concept, you have to first revise congruence of plane figures which students have met in grade 7. After revising this basic concept, you have to proceed to the notion of similarity. Finally, the knowledge of similarity is imparted through theorems presented. This knowledge of similarity should enable the students to work out some problems.

Learning outcomes: At the end of this unit, learners will be able to:

- ❖ Know the concept of similar figures and related terminologies.
- ❖ Understand the condition for triangles being similar.
- ❖ Apply tests to check whether two given triangles are similar.
- ❖ Apply real-life situations in solving geometric problems.
- ❖ Explain the concept of similar figures and related terminology
- ❖ Identify the condition for being similar

Suggested Teaching Aids

In addition to student's text book and teacher's guide, you are advised to bring ruler, colour chalk, maps, pictures, photographs, etc.

4.1. Similar plane figures

Period Allotted: 8 periods

4.1.1. Definition and illustration of similar figures

Competency: At the end of this sub-topic, students should:

- ❖ Identify figures that are similar to each other.

Introduction:

You may start the lesson by revising congruency of plane figures in that two figures are congruent if they have the same size and shape. Then you may ask the students to tell what happens if the figures do have the same shape but different size? Following their reply, you can start discussion by defining similar figures as "Similar figures are identical in shape but not necessarily in size". In other works, in a pair of similar figures, one shape is an enlargement of the other. After giving this preliminary information about the lesson, you precede your teaching by making the students participate in the teaching – learning process. In this particular lesson, you may use one of the active learning methods known as Drill

Partners. You group the students and let them drill each other by first asking them questions given in Activity 4.1 and questions of the following types.

Which of the following pairs are always similar? Any two squares, any two rectangles, any two right triangles.

Next to sharpen student intuition, let discuss the definition of similar figures: “For any pair of similar figures, corresponding sides have the same ratio and corresponding angles are congruent” as given in the student textbook. The main purpose of Activity 4.1 is to let students use their common sense or experience so as to identify similar figures. Based on their feedback and after making the necessary correction it is recommendable way to build the concept under discussion systematically. So let the students free to express their opinion and by asking them some leading question guide them to the concept of similarity.

Answers to Activity 4.1

1. A and C, B and E, D and G, F and H are similar figures.
2. No, because they do not necessarily fit together.
3. Yes, they are congruent by SAS congruence theorem.
4. They are congruent (by angle-sum theorem).
5. a) not similar

   b) not similar

Let some students present the answer of Activity 4.1. Then you can enrich their understanding by doing the examples given in the student textbook.

Assessment:

Students understand the given lesson differently at different level. Therefore, in order to ensure that all students can fully participate in their learning, you may give various techniques of assessments. You can give class activities, group discussion, exercise problems and quiz or test. These helps to check students how much they achieved.

Answers to Exercise 4.1

1a) True b) False c) True d) False e) False

2) $k = 1$

3) Let x, y, z be the remaining sides of the quadrilateral. Then

$$\frac{12}{x} = \frac{9}{y} = \frac{16}{z} = \frac{20}{8} = \frac{5}{2}$$

$$\frac{12}{x} = \frac{5}{2}, \qquad \frac{9}{y} = \frac{5}{2}, \qquad \frac{16}{z} = \frac{5}{2}$$

$$x = \frac{24}{5} \qquad y = \frac{18}{5} \qquad z = \frac{32}{5}$$

4.1.2. Similar Triangles

Competency: At the end of this sub – section students should:

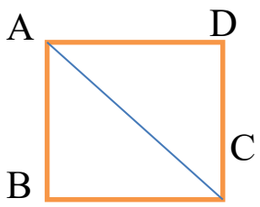
- ❖ Apply the definition of similarity of two triangles to solve related problems.

Now you have defined similar polygons in section 4.1.1. Also you know the students are familiar with "any polygon could be dividing into triangles by drawing the diagonals of the polygon". Thus the definition you gave for similar polygons could be used to define similar triangles. Stabilize what you defined so by working example 4.7 and 4.8 with active participation of the students.

Answers to Exercise 4.2

1a) $x =$ AB b)$x = \overline{FD}$ c) $y = \angle E$

2) Given square ABCD.



Statements	Reasons
1. $\angle B \cong \angle D$	Right angle
2. $\angle BAC \cong \angle DAC$	Diagonal of a square is an angle bisector.
3. $\angle ACB \cong \angle ADC$	Diagonal of a square is an angle bisector.
4. $\Delta ABC \cong ADC$	AA congruence theorem

Since every congruent triangle is similar, then $\Delta ABC \sim ADC$.

4. a) yes

b) Yes. For any triangle, there exists a similar triangle with any given constant of proportionality.

4.1.3. **Tests for similarity of triangles (AA, SSS, and SAS)**

Competency: At the end of this section, students should:

- ❖ Determine the similarity of two triangles.

Before you discuss the theorems on similarity; first you may remind the students the definition of similarity of triangles. Explain that two triangles are similar if their corresponding sides are proportional and their corresponding angles are congruent. That is, Δ ABC is similar to Δ DEF (Δ ABC ~ Δ DEF) if their Corresponding: Sides are proportional

[$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k$] and corresponding angles are congruent [$\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$]

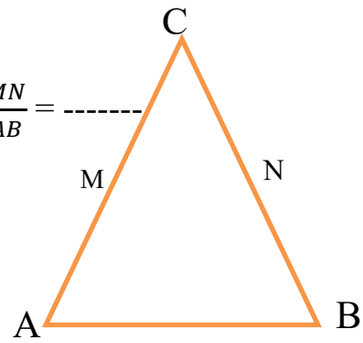
To prove similarity of triangles, using the definition of similarity means and checking all the six conditions required by the definition. This is long and tiresome. However, explain to the students that as with congruent triangles, there are special properties (tests) to use as shortcuts showing that two triangles are similar. In this lesson, these tests are known as tests for similarity [Similarity theorems].

You may start the lesson by stating and discussing AA, SAS, and SSS similarity theorems one by one. You have to make sure that students understand and hence can apply the theorems to solving problem and solving triangle. For this purpose, group the students and ask questions of the following types.

1. In the figure below, Δ ABC, the midpoint of $\overline{AC}$ is M and the midpoint of $\overline{BC}$ is N. Then answer the following questions

   a) What is the ratio: $\frac{CM}{MA}$ = -------, $\frac{CN}{CB}$ = -------, $\frac{MN}{AB}$ = -------

   b) Show that Δ ABC ~ Δ MNC.

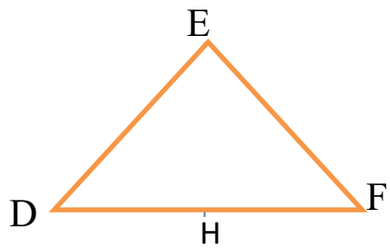


2. Given a right angle triangle DEF with right angle at E if a perpendicular is drawn from E as shown. Prove that

   a) Δ DHE ~ Δ DEF

   b) Δ FHE ~ Δ FED

   c) ΔDHE~ Δ EHF



While students are working on the answers, go round each group and you facilitate their discussion. After making them warmed up with the question of the above type, you can give the theorems discuss each theorems with examples given in the text. After you do examples, you need to summarise the similarity theorems (tests) and enrich each with additional examples.

Assessment

To assess students understanding there could be various ways that you may follow. However, to suggest them you can give them assignments to prove the tests. You can also give them exercise 4.3 as a home work to verify similarity by applying the similarity test.

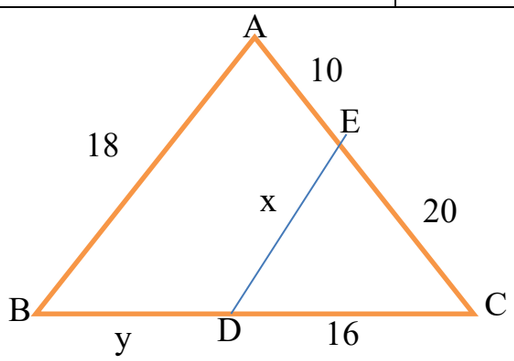
Answer to Activity 4.2

Yes, by AA similarity they are similar.

Answers to Exercise 4.3

1a)

Statements	Reasons
1. $\angle C \cong \angle C$	Common angle
2. $\angle B \cong \angle D$	Corresponding angles
3. $\Delta ACB \sim \Delta ECD$	AA similarity theorem



b. Since, $\Delta ACB \sim \Delta ECD$

$\frac{AC}{EC} = \frac{CB}{CD} = \frac{AB}{ED} = k$

$\frac{AC}{EC} = \frac{CB}{CD}$

$\frac{30}{20} = \frac{y+16}{16}$

$y + 16 = 24$

$y = 8$

$\frac{AC}{EC} = \frac{AB}{ED}$

$\frac{30}{20} = \frac{18}{x}$

$3x = 2 \times 18$

$x = 12$

2. $\Delta LMN \sim \Delta PQR$

$\frac{LM}{PQ} = \frac{MN}{QR} = \frac{LN}{PR} = k = \frac{2}{3}$

$\frac{12}{PQ} = \frac{2}{3}$

$PQ = 18$

$\frac{15}{QR} = \frac{2}{3}$

$QR = 22.5$

$\frac{18}{PR} = \frac{2}{3}$

$PR = 27$

Therefore, the lengths of the corresponding sides of ΔPQR are 18, 22.5, and 27.

3. $\frac{12}{18} = \frac{2}{3}$

4.a)

Statements	Reasons
1. $\angle RSQ \cong \angle RQT$	Right angles
2. $\angle R \cong \angle R$	Common angle
3. $\Delta RSQ \sim \Delta RQT$	AA similarity theorem

b. $\frac{RS}{RQ} = \frac{SQ}{QT} = \frac{RQ}{RT} = k$

$\frac{RQ}{RT} = \frac{15}{25} = k$

$k = \frac{3}{5}$

c. $\frac{SQ}{QT} = \frac{3}{5}$

$SQ = 12$

d. $\angle RSQ \cong \angle QST$ [right angle]

$\frac{RS}{QS} = \frac{9}{12} = \frac{3}{4}$, $\frac{QS}{ST} = \frac{12}{16} = \frac{3}{4}$

Since, the two corresponding sides have the same ratio and the included angle is congruent, by SAS $\angle RSQ \cong \angle QST$.

5. $\Delta PQR \sim \Delta LMN$

$\frac{PQ}{LM} = \frac{QR}{MN} = \frac{PR}{LM} = k$

$\frac{QR}{10} = \frac{3}{2}$ $\frac{18}{LM} = \frac{3}{2}$

$QR = 15$ $LM = 12$

6. Let the remaining sides of a triangle be x and y.

The scale factor of a triangle is $\frac{4}{12} = \frac{1}{3}$. Then

$\frac{9}{x} = \frac{1}{3}$ and $\frac{11}{y} = \frac{1}{3}$

$x = 27$ and $y = 33$

4.2. Perimeter and Area of Similar Triangles

Period Allotted: 6 periods

Competencies: At the end of this sub-topic, students should:

- Explain the relation between the perimeters of two similar triangles.
- Explain the relation between the areas of two similar triangles.

Introduction:

So far students have discussed similarity theorems that we can use to check whether two plane figures are similar or not. Before you start the teaching of the concept of the lesson, first you group the students and ask them to recall on their own for a couple of minutes and say that all they can remember about similar figures. Then ask them to discuss their response with students in their respective groups. After a few minute, ask some of the groups to share their common answers with the whole class. Having done this, you may start the lesson by asking students to do Activity 4.3. Assist each group to do the Activity according to the steps enlisted in the student text. Then discuss and give the correct generalization. Now we are going to see properties that prevail similar figures in the form of ratios of sides, areas, and perimeters and possible relationship between each.

Make sure that the students have understood theorem 4.4 and theorem 4.5.

Assign all problems given in Exercise 4.4 as class work and home work.

Answers to Activity 4.3

1) Perimeter of a triangle is the sum of the lengths of all three sides. Area of a triangle is a region bounded by a triangle.

2) A = $\frac{1}{2}$bh, where b and h are legs a right angled triangle.

3) a) P_1 = p(ΔRPQ) = 15+25+20 = 60

P_2 = p(ΔDEF) = 6+10+8 = 24

b) $\frac{P1}{P2} = \frac{60}{24} = \frac{5}{2}$

c) $\frac{RP}{DE} = \frac{15}{6} = \frac{5}{2}$

$\frac{P1}{P2} = \frac{s1}{s2} = \frac{5}{2}$

d) A_1 = a(ΔRPQ) = $\frac{1}{2}$ (15×20) = 150

A_2 = a(ΔDEF) = $\frac{1}{2}$ (6×8) = 24

e) $\frac{A1}{A2} = \frac{150}{24} = \frac{25}{4}$

f) $\frac{A1}{A2} = \frac{25}{4} = (\frac{5}{2})^2 = (\frac{s1}{s2})^2$

g) The ratio of the perimeter of two similar triangles is equal to the ratio of their corresponding sides. And

The ratio of area of two similar triangles is equal to the square of the ratio of their corresponding sides.

Answers to Exercise 4.4

1a. Since Δ ABE ~ Δ ACD by AA similarity

$\frac{AB}{AC} = \frac{BE}{CD}$ | $\frac{AE}{AD} = \frac{6}{9}$

$\frac{6}{6+y} = \frac{6}{9}$ | $\frac{x}{x+4} = \frac{6}{9}$

$y = 3$ | $x = 8$

Thus, Perimeter of Δ ABE = AB + BE + AE = 6 + 6 + 8 = 20 and

Perimeter of ΔACD = 9 + 9 + 12 = 30.

Therefore, $\frac{p(\Delta\,\mathrm{ABE})}{p(\Delta\mathrm{ACD})} = \frac{20}{30} = \frac{2}{3}$

b. $\frac{a(\Delta\,\mathrm{ABE})}{a(\Delta\mathrm{ACD})} = \left(\frac{S_1}{S_2}\right)^2 = \left(\frac{6}{9}\right)^2 = \frac{4}{9}$

2. $\frac{a(\Delta\,\mathrm{ABC})}{a(\Delta\,DEF)} = k^2$

$\frac{x}{x-7} = \left(\frac{4}{3}\right)^2$

$9x = 16x - 112$

$x = 16$

$\frac{p(\Delta\ ABC\ )}{p(\Delta\ DEF)} = k$

$\frac{8+y}{3y-12} = \frac{4}{3}$

$24 + 3y = 12y - 48$

$y = 8$

a) $p(\Delta\ DEF) = 3y - 12 = 3 \times 8 - 12 = 12$ units

b) $a(\Delta\ ABC\ ) = x = 16$ Square unit.

3) let x be the length of altitude $\overline{FQ}$, then $x + 7$ is the length of altitude $\overline{BP}$.

Applying AA similarity theorem gives

$\frac{x+7}{x} = \frac{AC}{DF}$

$\frac{x+7}{x} = \frac{5}{4}$

$4x + 28 = 5x$

$28 = x$

Therefore, altitude FQ = 28

Altitude BP = 28 + 7 = 35

Assessment

For slow learners

1) ΔABC~ ΔDEF. Name three pairs of corresponding angles and three equal ratios.

For fast learners

1) ΔRST~ ΔJKL. The ratio of altitude of ΔRST to ΔJKL is 3: x. The length of altitude $\overline{SU}$ is $x - 4$ and the length of altitude $\overline{ku}$ is 15. Find the length altitude $\overline{SU}$.

Answers to Review Exercise

1a) True b) False c) True

d) True e) False f) False

2) $\Delta\ ABC \sim\ \Delta\ DEF$

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{3}$

$\frac{3}{DE} = \frac{1}{3}$

$DE\ = 9$

$\frac{BC}{EF} = \frac{1}{3}$

$\frac{5}{EF} = \frac{1}{3}$

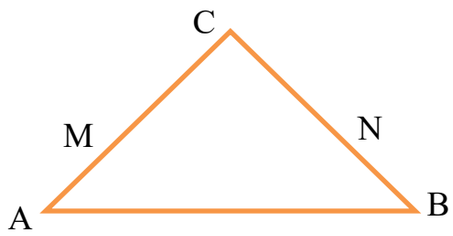
$EF = 15$

$\frac{AC}{DF} = \frac{1}{3}$

$\frac{7}{DF} = \frac{1}{3}$

$DF\ = 21$

3a)



$\angle C \cong \angle C$ ----- common angle

$\frac{BC}{NC} = \frac{AC}{MC}$

Therefore, Δ ABC ~ Δ MNC by SAS similarity theorem.

b) $\frac{1}{2}$

4) Δ ABC ~ Δ DBE by AA similarity theorem.

$\frac{AB}{DB} = \frac{BC}{BE} = \frac{AC}{DE}$

$\frac{x+30}{30} = \frac{22}{14}$

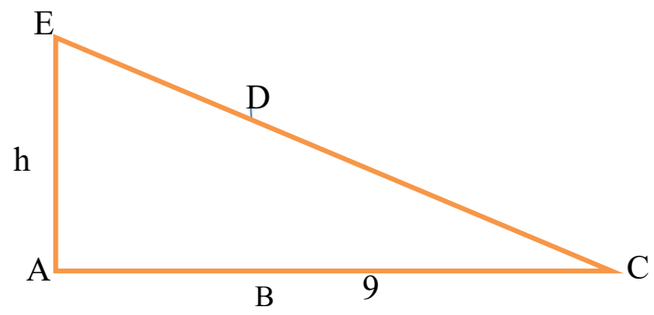
$7(x + 30) = 330$

$x = 17.14$

$\frac{15+y}{y} = \frac{22}{14}$

$14(15 + y) = 22y$

$y = 26.25$



5) ΔACE ~ ΔBCD by AA similarity theorem.

$\frac{AC}{BC} = \frac{AE}{BD}$

$\frac{26}{9} = \frac{h}{1.8}$

$h = 5.2$

Therefore, the height of the tree is 5.2 metre.

6) Parallelogram

$\angle BEC \cong \angle DEA$ ------ vertical angles

$\angle CBD \cong \angle ADB$------- alternate interior angles

ΔBEC ~ ΔDEA by AA similarity theorem

7) $\angle A \cong \angle D$ and $\angle C \cong \angle F$ ------- corresponding angles.

8) $\angle A \cong \angle A$ ----- common angle, $\frac{AD}{AB} = \frac{AE}{AC}$

By SAS similarity theorem, Δ ADE ~ Δ ABC

9) $\frac{AD}{AB} = \frac{DE}{BC}$ and $\frac{AE}{AC}$

10) $\angle E \cong \angle G$--------- right angles.

$\angle B \cong \angle D$------- Opposite angles of a parallelogram are congruent.

Therefore, by AA similarity theorem, Δ AED ~ Δ FG

UNIT 5

THEOREM ON TRIANGLES

Total Allotted period: 15 periods

LEARNING OUT COMES: At the end of this unit, learners will be able to:

- Understand basic concepts about right angled triangles
- Apply some important theorems on right angled triangles.
- Apply real-life situations in solving geometric problems
- Explain the basic concepts of right angled triangle

Introduction

The students are already familiar with triangle and its parts. It will be better if you revise the basic congruency and similarity concepts and theorems. In this unit we will discover more interesting properties of a right –angled triangle though the relationship between the legs and hypotenuse. A right – angled triangle is a type of triangle that has one of its angles equal to 90 degree. The other two angles add up to 90 degrees. The sides that include the right angle are perpendicular and the base of right angle. The third sides is called hypotenuse, which is the longest of all three sides. The most important relationship for triangle is Pythagoras theorem. Before you start discussions about theorems on triangle activities should be done so the students can understand the theorems on triangles.

Suggested Teaching Aids in Unit 5

You can encourage your students work to draw any triangle, cut it out carefully, tear the vertices off and fit them together.

Based on the result, the sum of the measures of interior angles of a triangle is 180°.

You can use the real model of triangle to present clearly understanding to your students and use geometry like ruler, compass, Set Square and protractor for this unit model of triangles. Group the students so that they can produce either real models or diagrammatic models on triangles. Each student is expected to participate in the construction activities.

5.1 The three angles of a triangle add up to 180°

Period Allotted: (2 periods)

Competency: At the end of this section, students should:

- Describe the angle sum theorem of a triangle.

- ❖ Apply the angle sum theorem of a triangle in solving related problems.

Introduction

In geometry, one of the most used shapes is a triangle. A triangle has three sides and three angles. These sides and angles are the elements of the triangle. All the polygons have two types of angles which are interior angles and exterior angles. As the triangle is the smallest polygon, it has three interior angles and six exterior angles. A triangle with vertices A, B, C is denoted by ΔABC. There are various kinds of triangles with different angles and edges, but, all of them follow the triangle sum properties. The two most important properties are the angle sum property of a triangle and the exterior angle property of a triangle.

Angle Sum Property of a Triangle

This property states the sum of the interior angles of a triangle is 180 degrees. Interior angles are formed at the vertex where any two edges of a triangle join. The angle between two sides of a triangle is called the interior angle. It is also known as the interior angle property of a triangle. This property states that the sum of all the interior angles of a triangle is 180°. If the triangle is ΔABC, the angle sum property formula is $\angle A+\angle B+\angle C = 180°$.

Answer for Group work 5.1

1.

a. Vertically opposite angles: Two angles that are a cross from each other. These angles are congruent.
b. Alternate interior angles: Two angles that are on opposite sides of the transversal and insides the parallel lines. These angles are congruent.
c. Alternate Exterior angles: Two angles that are on opposite sides of transversal and out sides the parallel lines. These angles are congruent.
d. Corresponding angles: Any pair of angles each of which is on the same side of one of two lines cut by transversal and on the same side if the transversal.

2. $60° + a = 180°$(Straight angle)

$a = 120°$

$b = 60°$ By vertically opposite angles

$a = c$ By Vertically opposite angles

Therefore, $c = 120°$

$b = d$ By alternate interior angles

Therefore, $d = 60°$

$c = e$ By alternate interior angles

Therefore, $e = 60°$

$a = g$ By alternate Exterior angles

Therefore, $g = 120°$

$60° = f$ By alternate Exterior angles

Therefore, $f = 60°$

3.

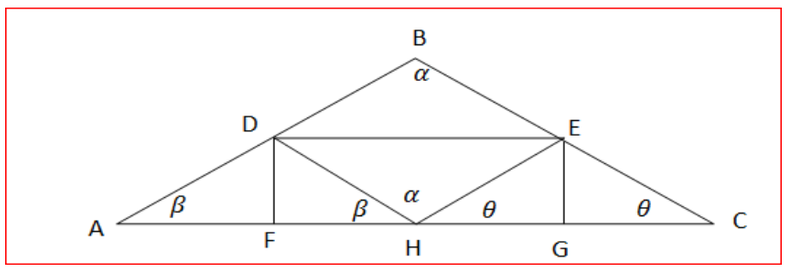


Figure 5.1

4. a. $x + 60° + 70° = 180°$

$$x + 130° = 180°$$
$$x = 180° - 130°$$
$$x = 50°$$

b. $\angle B = 111.76° \approx 112°$

c. $x = 60°$

Answer for exercise 5.1

1. a. m∠ABC + m∠BCA + m∠CAB = 180^0

$$2x + 75° + 25° = 180°$$
$$2x + 100° = 180°$$
$$2x = 180° - 100°$$
$$2x = 80°$$
$$x = 40°$$

b. m∠ABC + m∠BCA + m∠CAB = 180^0

$$90° + x + 35° = 180°$$
$$x + 125° = 180°$$
$$x = 55°$$

2. $2x + 3x + 4x = 180°$

$$9x = 180°$$
$$x = 20°$$

Therefore, the largest interior angle of a triangle is $4x = 80°$

3. In a right triangle we know that one of the angles is 90°. We also know that the 3 angles must add up to 180°.

Let x = acute angle 1

y = acute angle 2

We have 2 unknowns; therefore, we must have 2 equations to solve this.

Eq. I Eq. II

90 + x + y = 180 x = 2(y + 3)

x + y = 180 - 90 x = 2y + 6

x + y = 90

Using the value for x in terms of y from Eq. II in Eq. I we can solve for y:

x = 2y + 6 Eq. II

x + y = 90 Eq. I

2y + 6 + y =90 substitute 2y + 6 for x

3y = 90 - 6 add the y terms and subtract 6 from both sides

3y = 84 simplify

y = 84/3 divide both sides by 3

y = 28 solve for y

Using this value for y in Eq. II we can solve for x:

x = 2y + 6 Eq. II

x = 2(28) + 6 substitute 28 for y

x = 56 + 6 simplify

x = 62 solve for x

Therefore, you get the other 2 acute angles as 28° & 62°.

4. $x = 60°, y = 120°$ and $z = 60°$
5. a. $m\angle 1 = 153°, m\angle 2 = 27°$,and $m\angle 3 = 95°$

   b. $\angle 1 = 42°, m\angle 2 = 78°, m\angle 4 = 108°$, $m\angle 3 = 30°$and $m\angle 5 = 68°$

Assessment

- Give exercise on angle sum theorems

-Ask students to describe the angle sum theorem of a triangle

-Ask students to prove the sum of the measures of interior angle of a triangle is 180°

-Ask students to solve problems by using the angle sum theorem of a triangle

- In addition to this for slow learners and fast learners, you can also give the following additional exercise problems

For slow learner students

1. What is the measure of ∠ B in the figure below?

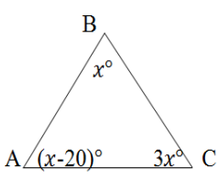
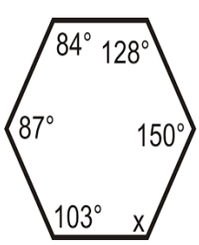
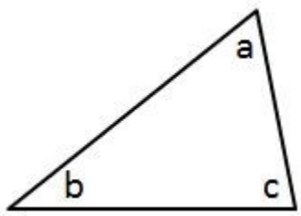


Figure 5.1

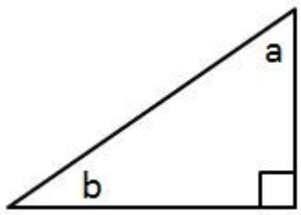
2. Find the value of x



3. If angle *a* is 68 degrees and angle *b* is 37 degrees, what is the measure of angle *c*?



4. If angle a is 60 degrees, what is the measure of angle b?



For fast learner students

2. The Ratio of the Three Angles of a Triangle is 1:2:3. Determine the Largest Angle of the Triangle and the Type of the Triangle.
3. In a triangle, the ratio between the first and second angle is 1 : 2 and the third angle is 72. Find the first and second angle of the triangle.
4. In a triangle, if the second angle is 3 times the sum of the first angle and 3 and the third angle is the sum of 2 times the first angle and 3, find the three angles of the triangle.
5. In a right triangle, apart from the right angle, the other two angles are x + 1 and 2x + 5. find the angles of the triangle.
6. In a triangle, if the second angle is 2 times the first angle and the third angle is 3 times the first angle, find the angles of the triangle.
7. If 3 consecutive positive integers be the angles of a triangle, then find the three angles of the triangle.
8. In a triangle, If the second angle is 20% more than the first angle and the third angle is 20% less than the first angle, then find the three angles of the triangle.
9. If the angles of a triangle are in the ratio 2 : 7 : 11, then find the angles.
10. In a triangle, If the second angle is 5° greater than the first angle and the third angle is 5° greater than second angle, find the three angles of the triangle.

5.2 The exterior angle of a triangle equals the sum of the two remote interior angles

Period Allotted: (3periods)

Competency: At the end of this section, students should:

- ❖ Describe the relation between the exterior angle and the two remote interior angles of a triangle.
- ❖ Prove the exterior angle of a triangle equals the sum of the two remote angles

Introduction

Exterior angle is the angle between one side of the triangle and the extended adjacent side. A triangle has six exterior angles. Each edge of a triangle can form two exterior angles with the two of its extended adjacent sides. If you want to get an exterior angle of a triangle, you have to extend the straight line of one side of the triangle. The straight line can be extended to any of the two sides. After the extension, the extended part and the adjacent side of the triangle form an angle. Thus, the exterior angle of a triangle is formed. The exterior angles have some properties as well. The exterior angle property is that the exterior angle adjacent to the largest exterior angle measures the smallest. The angles of a triangle opposite to the equal sides are equal. A triangle cannot have two or more right angles.

Answer for Activity 5.1

1. Straight angle**:** an angle whose sides lie in opposite directions from the vertex in the same straight line and which equals 180^0
2. The exterior angle of a triangle is the angle formed between one side of a triangle and the extension of its adjacent side.
3. Rules to find the exterior angles of a triangle are pretty similar to the rules to find the interior angles. It is because wherever there is an exterior angle, there is an interior angle with it, and both add up to 180 degrees.
4. Proof: let ABC be a triangle with $\overline{AC}$ extended to form an exterior angle. Let $a, b \; and \; c$ be degree measures of the interior angles of a triangle ABC and d be degree measure of exterior angle.

We want to show that: $a + b = d$

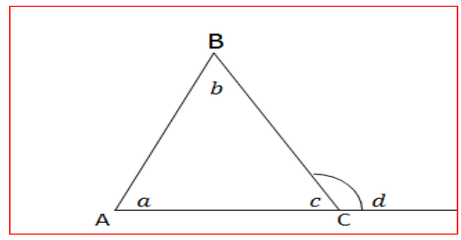


Figure 5.3

Statements	Reason
1. $a + d = 180°$	1. Definition of straight angle
2. $a + b + c = 180°$	2. Angle sum theorem
3. $a + b + c = a + d$	3. Substitution
4. $a + b = d$	4. Subtracting c from both sides

Answer for exercise 5.2

1. $x = 5°$
2. a) $x = 88°$ and $y = 47°$

   b. $x = 70°, y = 80°$ and $z = 50°$

   c. $x = 65°$

2 Definition of straight angle

3 m∠ABC + m∠BCA + m∠CAB

4 Substitution

5 Subtracting $m\angle$ACB from both sides

Assessment

You should assess each student's work continuously over the whole periods and discuss it with the following points

- Ask students to describe the exterior angle and the two remote interior angles of a triangle
- Ask students to prove the exterior angle of a triangle equals the sum of the two remote interior angles

- Ask students to solve problems using the exterior angle theorem of triangle
- Give informal assessment techniques like group work, class work, homework, oral and written questions, assignments, quizzes, tests etc. during each period
- Finally give additional exercise problems

For slow learner students

(I)Write true if the statement is correct and false if it is not.

1. Exterior angles of a triangle are never acute
2. An exterior angle of a triangle may be equal to one of the interior angles
3. An exterior angle of a triangle may be smaller than one of the interior angles
4. all of the exterior angles of a triangle may be obtuse
5. all of the exterior angles of a triangle may be acute
6. The sum of the exterior angles is 360°
7. An exterior angle is supplementary to its adjacent interior angle
8. if the interior angle is obtuse, the exterior angle will be acute
9. Equilateral triangle, all interior angles are 60°, all exterior angles are 120°

(II)Answer the following question

10. Find out the Angle ∠ABC of the Triangle ΔABC. The Exterior ∠ACD = 125° and the Other Interior Angle ∠BAC = 61°.
11. Calculate values of x in the following triangle.

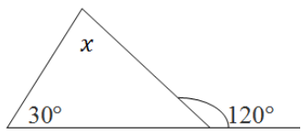
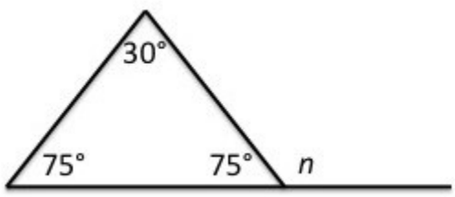


Figure 5.2

12. Solve for the missing exterior angle, *n*.



13. The exterior angle of a triangle is 120°. Find the value of x if the opposite non-adjacent interior angles are (4x + 40) ° and 60°.

14. Find the measure of the longest angle of a triangle of the measures of its interior angles are in the ratio 3:5:7.

For fast learner students

(III)Write true if the statement is correct and false if it is not.

15. An exterior angle of a triangle equals 180° minus the adjacent interior angle
16. An exterior angle of a triangle is equal to the sum of the two remote angles
17. An exterior angle of a triangle is less than either of its interior opposite angles
18. Find the degree measures of marked angles in figure5.3 below (the letters a-h represent degree measures of the angles

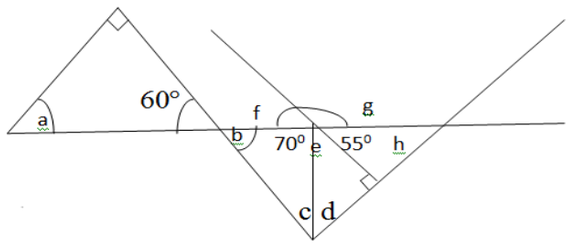


Figure 5.5

19. Find the values of x and y in the following triangle.

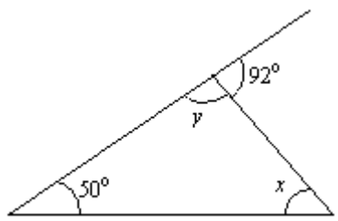
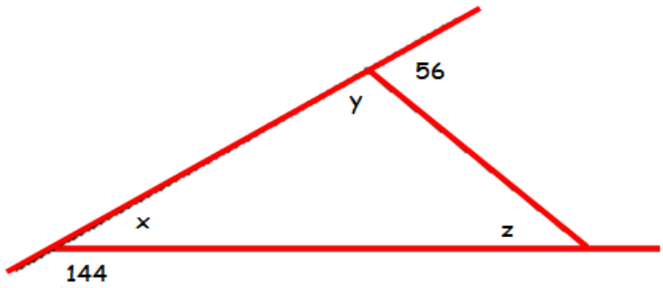


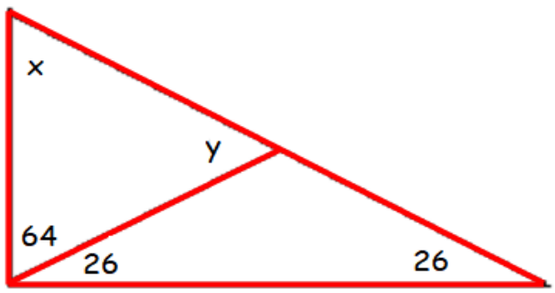
Figure 5.6

20. In triangle PQR, the measure of <P is 36°. The measure of <Q is five times the measure of <R. Find <Q and <R
21. Find the value of x, y and z

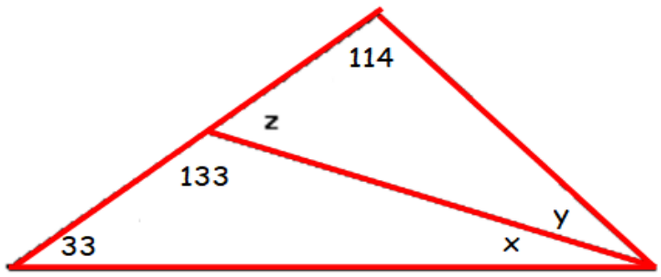
a)



b)



c)



22. The measure of one angle of a triangle is 3 times the measure of the smallest angle. The measure of the third angle is 25 degree morthan the measure of the smallest angle Find the measure of each angle.
23. Find the measure of the vertex angle of an issoscles triangle if the measure of the vertex angle and a base angle have the ratio 4:3.

5.3 Theorems on the right angled triangle

Total Allotted period: 10 periods

5.3.1 Euclid' Theorem and its Converse

Competency: At the end of this section, students should:

- Describe the right angle triangle, the altitude and hypotenuse
- Apply Euclid's theorem and its converse for solving related problems

Introduction

A right-angled triangle is a type of triangle that has one of its angles equal to 90 degrees. The other two angles sum up to 90 degrees. The sides that include the right angle are perpendicular and the base of the triangle. The third side is called the hypotenuse, which is the longest side of all three sides. The side opposite to the right angle is the smallest side. The three sides of the right triangle are related to each other. This relationship is explained by Pythagoras theorem. According to this theorem, in a right triangle, Hypotenuse^2 = Leg^2 + Leg^2

Before you start discussions about theorems on triangles activities and group works should be done so the students can understand the theorems on triangles. Depending the concepts of right angle triangle, students should practice using Activity 5.2 of the text book.

Answer for Activity 5.2

1. $\overline{DC}$ is altitude, $\overline{AB}$ is hypotenuse and ΔACB is a right angled triangle
2. ΔADC~ΔBDC

   ΔACB~ΔADC

   ΔACB~ΔBDC

After giving a brief summary of what has been discussed so far and giving students enough time to relate the basic facts, definition, examples and theorems give Exercise 5.3 as a home work and give marks and record it.

Answer for Exercise 5.3

1.

a. AB = AD + BD = 4cm + 8cm = 12cm
b. $(\text{BC})^2 = \text{DB} \times \text{AB}$…… Euclid's Theorem

$$(\text{BC})^2 = 8\text{cm} \times 12\text{cm}$$

$$\text{BC}^2 = 96\text{cm}^2$$

$$BC = \sqrt{96cm^2}$$

$$BC = 4\sqrt{6}cm$$

c. $(\text{AC})^2 = \text{AD} \times \text{AB}$…… Euclid's Theorem

$$(\text{AC})^2 = 4\text{cm} \times 12\text{cm}$$

$$\text{AC}^2 = 48\text{cm}^2$$

$$AC = \sqrt{48cm^2}$$

$$AC = 4\sqrt{3}cm$$

d. $CD^2 + AD^2 = AC^2$

$$CD^2 + (4cm)^2 = (4\sqrt{3}cm)^2$$

$$CD^2 + 16cm^2 = 48cm^2$$

$$CD^2 = 48cm^2 - 16cm^2$$

$$CD^2 = 32cm^2$$

$$CD = \sqrt{32cm^2}$$

$$CD = 4\sqrt{2}cm$$

2. $AD^2 + DB^2 = AB^2$

$$(4cm)^2 + (8cm)^2 = AB^2$$

$$16cm^2 + 64cm^2 = AB^2$$

$$AB = 4\sqrt{5}cm$$

$$c = 4\sqrt{5}cm$$

$$(\mathrm{BC})^2 = \mathrm{AC} \times \mathrm{DC}$$

$$(\mathrm{BC})^2 = 24\mathrm{cm} \times 15\mathrm{cm}$$

$$\mathrm{BC}^2 = 360\mathrm{cm}^2$$

$$BC = \sqrt{360cm^2}$$

$$BC = 6\sqrt{10}cm$$

$$a = 6\sqrt{10}cm$$

$$(\mathrm{AB})^2 = \mathrm{AD} \times \mathrm{AC}$$

$$(\mathrm{AB})^2 = 4\mathrm{cm}(\mathrm{x} + 4\mathrm{cm})$$

$$(4\sqrt{5}\mathrm{cm})^2 = 4\mathrm{xcm} + 16\mathrm{m}^2)$$

$$4xcm = 80cm^2 - 16cm^2$$

$$4xcm = 64cm^2$$

$$x = 15cm$$

3. a) The altitude if an equilateral triangle with 4cm is $2\sqrt{3}cm$

   b) The altitude if an equilateral triangle with 6cm is $2\sqrt{11}cm$

4. ΔABC is a right angled triangle.

Assessment

❖ Give exercise on the Euclid's Theorem and its application

Ask students to describe the right angle triangle, the altitude and hypotenuse

Ask students to solve problems using Euclid's theorem and its converse

Provide descriptive feedback for learners to improve their learning

Additionally, you can also give the following questions depending on the students' performances

For slow learner students

(I)Write true if the statement is correct and false if it is not.

1. Euclidean geometry is geometry on a prism
2. All geometry is Euclidean
3. In Euclidean geometry: there is only one equilateral triangle
4. The rules of parallel and perpendicular lines are different in spherical geometry than in Euclidean
5. Euclidean geometry is valid only for curved surface

(II)Short answer (show necessary steps)

6. Find the value of $x, y$and z

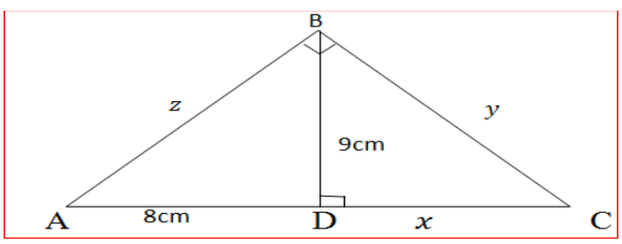


Figure 5.7

7. ΔABC is a right- angled triangle as shown in the diagram. If $\overline{AD}$= 12cm and $\overline{BD}$ = $\overline{DC}$ then find the length of $\overline{BD}$ and $\overline{DC}$

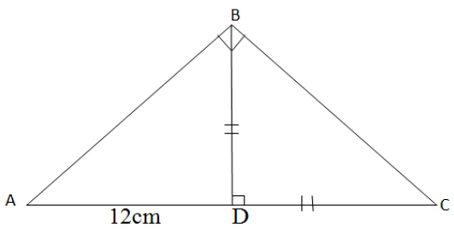


Figure 5.8

For fast learner students

8. How long is an altitude of an equilateral triangle if a side of a triangle is:

a. 6cm b. 8cm

9. In figure 5.6 below, ΔABC is right angled at B,

$\overline{BD} \perp \overline{AC}$, $\overline{BE} = \overline{BC}$, $\overline{BE}$ = 6cm, $\overline{AC}$= 12cm. Find

a. $\overline{BC}$

b. $\overline{DC}$

c. $\overline{AB}$

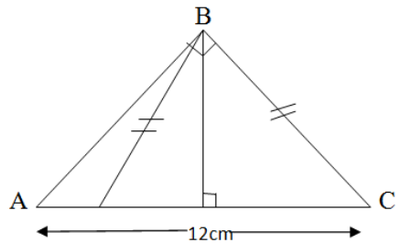


Figure 5.9

10. A triangle has a leg of 5cm, height of 12cm and a hypotenuse of 13cm. Is the triangle right – angled?

5.3.2 The Pythagoras' theorem and its converse

Competency: At the end of this section, students should:

- ❖ Derive the Pythagoras theorem by using Euclid's theorem and paper folding
- ❖ Apply Pythagoras' Theorem and its converse for solving related problem.

Introduction

The Pythagorean Theorem relates the lengths of the three sides of any right angle triangle. The two sides of the triangle that meet at the right angle are normally labeled as having lengths "a" and "b", and the hypotenuse, which is the long side of the triangle, opposite the right angle, is labeled with the variable "c". The Pythagorean Theorem relates the side lengths according to the following equation: $a^2 + b^2 = c^2$. This theorem allows you to find the length of the third side of a triangle if you know the first two side lengths and you know you are dealing with a right angle triangle.

Answer for Activity 5.3

1. Euclid's Theorem state that in a right angled triangle with an altitude to the hypotenuse, the square of the length of each leg of a triangle is equal to the product of the hypotenuse and the length of the adjacent segment into which the altitude divides the hypotenuse.

2. a, b and d are right triangle, but c is note right triangle

3. Algebraic Method proof of Pythagoras theorem

Algebraic Method proof of Pythagoras theorem will help us in deriving the proof of the Pythagoras theorem by using of a, b and c (values of the measures of the sides corresponding to sides BC, AC, and AB respectively)

Consider four right triangle ABC where b is base, a is altitude and c is hypotenuse. Arrange these four congruent right triangles in the given square, whose side is $a + b$.

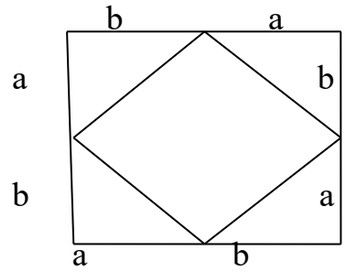


Figure 5.10

The area of the square so formed by arranging the four triangles isc^2.

The area of a square with side $(a+b)$ =Area of 4 triangles + Area of square with side c

This implies $(a+b)^2 = 4 \times \frac{1}{2}(a \times b) + c^2$ but

$$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$$

$$a^2 + b^2 + 2ab = 2ab + c^2$$

$$a^2 + b^2 + 2ab - 2ab = 2ab - 2ab + c^2$$

Therefore, $a^2 + b^2 = c^2$ hence proved

Pythagoras theorem proof using similar triangles

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides. That is legs of lengths$a$ and b hypotenuse of length c,then $a^2 + b^2 = c^2$

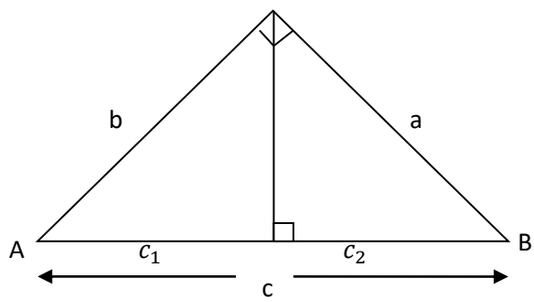


Figure 5.11

Proof: Let ΔABC be right angled triangle the right angle at C as shown above.

Given: ΔACB is a right triangle and $\overline{CD} \perp \overline{AB}$.

We want to show that:$a^2 + b^2 = c^2$

Statements	Reason
1. $a^2 = c_2 \times c$	1. Euclid's Theorem
2. $b^2 = c_1 \times c$	2. Euclid's Theorem
3. $a^2 + b^2 = (c_2 \times c) + (c_1 \times c)$	3. Adding step1and step2
4. $a^2 + b^2 = c(c_2 + c_1)$	4. Taking c as a common factor

5. $a^2 + b^2 = c(c)$ 5. Since $(c_2 + c_1) = c$
6. $a^2 + b^2 = c^2$ 6. proved

4. Pythagorean triple consists of three positive integers a, b and c which satisfy $a^2 + b^2 = c^2$

a. When $a = 15$ and $b = 20$,then find the value of c

b. When $a = 24$ and $c = 40$,then find the value of b

Solution: Apply Pythagorean triple

a. $a^2 + b^2 = c^2$

$(15)^2 + (20)^2 = c^2$

$225 + 400 = c^2$

$c^2 = 625$

$c = \sqrt{625}$

$c = 5$

b. $a^2 + b^2 = c^2$

$(24)^2 + b^2 = (40)^2$

$576 + b^2 = 1600^2$

$b^2 = 1600 - 576$

$c^2 = 1024$

$c = \sqrt{1024}$

$c = 32$

5. The three numbers that satisfy Pythagorean Theorem are $\{6,8,10\}$,$\{9,12,15\}$ and $\{12,16,20\}$ in general patterns of Pythagorean triple is $\{3x, 4x, 5x\}$ respectively for x is a positive integers.

After giving a brief summary of what has been discussed so far and giving students enough time to relate the basic facts, definition, theorem and examples give exercise 5.4 as home work.

Answer for Exercise 5.4

1. In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides. That is legs of lengths$a$ and b hypotenuse of length c,then $a^2 + b^2 = c^2$

2. The length sides of rhombus are 5cm.

3. a. $x = 5cm$ b. $x = 8cm$ c. $x = 12cm$

4. The length of the hypotenuse is $45cm$

5. Let x be each length sides of square

   $x^2 + x^2 = (12cm)^2$

   $2x^2 = 144cm^2$

   $x = 6\sqrt{2}cm$

6. The height of the top of the ladder is 12meter.

7. The distance between the man and starting point is $26km$.

8. i) a. $\overline{AE} = 4cm$

   b. $\overline{DF} = 4cm$

   c. $\overline{EF} = 6cm$

   d. $\overline{AB} = \overline{AD} + \overline{DB}$

$\overline{AB} = 6cm + 3cm = 9cm$

ii) Yes ΔCEF is a right triangle

iii) Yes ΔBDF is a right triangle

Assessment

You should asses each student's work continuously over the whole period and discuss it the following questions

- ❖ Ask students to derive the Pythagoras theorem by using Euclid's theorem and paper folding
- ❖ Ask students to solve problems using Pythagoras' Theorem and its converse
- ❖ Additionally, you can also give the following questions depending on the students' performances

For slow learner students

Answer the following questions

(I)Write true if the statement is correct and false if it is not.

1. The Pythagorean Theorem can be used to find missing angle measures in a right triangle
2. The Pythagorean Theorem can be used to find missing side length in a right triangle
3. The Pythagorean Theorem can be used to find missing side length in any triangle
4. The side length of a triangle should always sum to 180
5. The side length of a right triangle should always sum to 90
6. A triangle with side length 5, 12 and 13 make right triangle

(II) Choose the correct answer from the given question

7. In a right triangle, the hypotenuse is always

   A) Next to the right angle

   B) The longest side

   C) The shortest side

   D) All

8. Pythagorean theorem is known as

   A) $a + b = c$

   B) $a^2 + b^2 = c^2$

   C) $a^2 + c^2 = b^2$

   D) $a^2 + b^2 = c$

9. In the Pythagorean Theorem, $a^2 + b^2 = c^2$, the variable c is

   A) The hypotenuse

   B) Next to the right angle

   C) A leg

   D) The shortest side

10. Which side of the triangle should be labeled as $'c'$

    A, The base B, The height C, The hypotenuse

11. Pythagoras' theorem will work on which type of triangles?

    A, Right- angled

    B, Isosceles triangle

    C, Any type of triangle

12. Calculate the length of x

a.

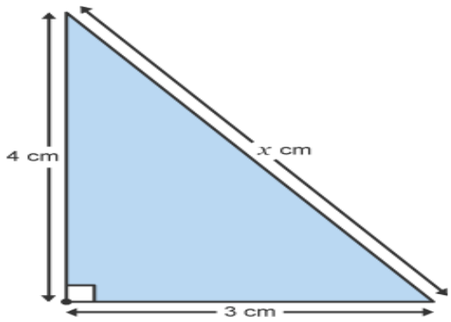


Figure 5.12

b.

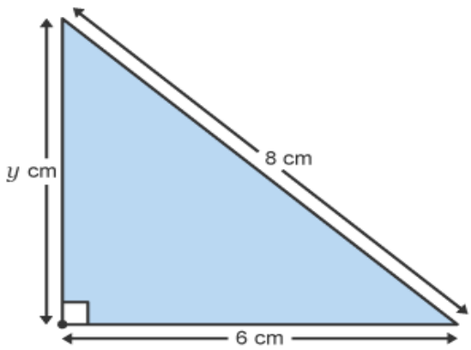


Figure 5.13

For fast learner students

13. Find the length of the altitude drawn to a side of an equilateral triangle whose perimeter is 30m?
14. If the perimeter of a square is 24, find the length of a diagonal.
15. A man goes 10 cm due East and then 24cm due North. Find the distance from the starting point.
16. The diagonal of a rhombus are 18 and 24. Find the length side of the rhombus.
17. The length of each leg of an isosceles trapezoid is 17cm. The lengths of its bases are 9 and 39. Find the length of an altitude.

Answer to Review Exercise

1) $6x + 8x + 10x = 180^0$

   $24x = 180^0$

   $x = 7.5$

   The measure of a triangle is 45^0, 60^0, and 75^0

2) $37^0+90^0+x = 180^0$

   $127^0 + x = 180^0$

   $x = 180^0 - 127^0 = 53^0$

3) Consider the following figure:

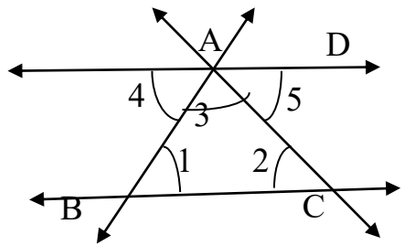


Figure 5.14

Given: ΔABC with $\overline{AD} \parallel \overline{BC}$

Prove:$m\angle 1 + m\angle 2 + m\angle 3 = 180$

Statements	Reasons
1. ΔABC With $\overline{AD} \parallel \overline{BC}$	Given
2. $\angle 4 \cong \angle 1,\ \angle 5 \cong \angle 2$	Alternate interior angles
3. $m\angle 4 + m(\angle BAD) = 180^0$	Definition of straight angle.
4. $m\angle 3 + m\angle 5 = m(\angle BAD)$	Addition
5. $m\angle 4 + m\angle 3 + m\angle 5 = 180^0$	Substitution
6. $m\angle 1 + m\angle 2 + m\angle 3 = 180^0$	Substitution

4) 50^0+80^0 + x = 180^0 ……..angle sum theorem

x = 180^0– 130^0 = 50^0

5) $(2x + 10)^0$ + $(4x)^0$ + $(3x)^0$ = 180^0

$9x$ = 180^0−10^0

x= 18.889^0

Therefore, the measure of angle B is 56.667^0

6) ΔABC is an isosceles triangle, an angle which are opposite to the congruent sides are congruent.

$m\angle A + m\angle B + m\angle C = 180^0$

$m\angle A + m\angle B + m\angle A = 180^0$

2 $m\angle A$ = 180^0− 40^0

$m\angle A$ = 70^0

7) Proof: Let ABC be a triangle with $\overline{AC}$ extended to form an exterior angle. Let x, y and z be the degree measures of the interior angles of Δ ABC and d be degree measure of the formed exterior angle.

We want to show that: $x + y = d$

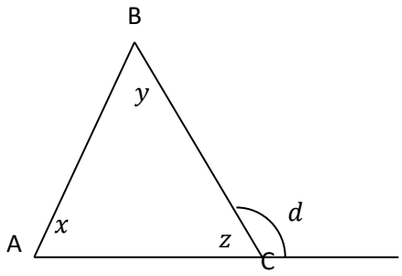


Figure 5.14

Statements	Reasons
1. $z + d = 180°$	Straight angle
2. $x + y + z = 180°$	Angle Sum Theorem
3. $x + y + z = z + d$	Substitution
4. $x + y = d$	Subtracting z from both sides

Therefore, the exterior angle of a triangle is equal to the sum of the two remote interior angles.

8) $(x + 10)^0 + (x + 20)^0 = (6x - 30)^0$

$(2x + 30)^0 = (6x - 30)^0$

$4x = 60^0$

$x = 15$

9) $x + 120^0 = 180^0$ ……. definition of straight angle

$x = 180^0 - 120^0 = 60^0$

$y + 40^0 = 120^0$ ……. Theorem 5.2

$y = 120^0 - 40^0 = 80^0$

10) Pythagoras theorem state as

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. That is, if legs of

lengths a and b, hypotenuse of length c,then $a^2 + b^2 = c^2$

11) Euclid's theorem state as

In a right angled triangle with an altitude to the hypotenuse, the square of the length of each leg of a triangle is equal to the product of the hypotenuse and the length of the adjacent segment into which the altitude divides the hypotenuse.

12) a) AB = AD + DB

= 8cm + 4cm

= 12cm

b) $(BC)^2$= BD ×AB

= 8 × 12

= 96

BC = $4\sqrt{5}$ cm

c) $(AC)^2$= AD ×AB

= 4 × 12

= 48

AC = $4\sqrt{3}$ cm

d) $(DC)^2 + (DB)^2 = (BC)^2$

$(DC)^2$ = 96 – 64

= 32

DC = $4\sqrt{2}$ cm

13) $(16)^2 + (48)^2$? $(50)^2$

256 + 2304 ? 2500

2560 ≠ 2500

Therefore, it is not a right angled triangle.

14) $(x)^2 + (x)^2$= 36

$2(x)^2$ = 36

x^2 = 18

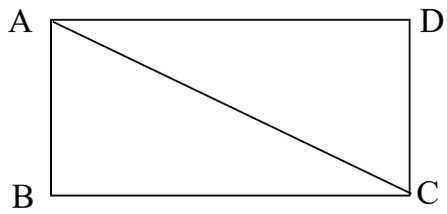
x = $3\sqrt{2}$ cm

15) Let x be the length from the foot to the top of tree strike the ground

x^2 =$(15)^2 - (3)^2$

x^2 = 216, x =

16) ABCD be a rectangle with diagonal AC



ΔABC is a right angled triangle, then

$(AC)^2 = (AB)^2 + (BC)^2$

$= (5)^2 + (12)^2$

$= 25 + 144$

$= 169$

AC = 13

17) $(c)^2 = (9)^2 + (12)^2$

$= 81 + 144$

$= 225$

C = 15

UNIT 6

LINES AND ANGLES IN A CIRCLE

Total Allotted Period: 17 periods

INTRODUCTION

The main task of this unit is to concern the different parts of the circle and what they are called, the angle and the relationship of angles and chords. Moreover, the theorems we will discuss will deal with the ways in which angles are measured by the arcs that they intercept. This unit divides into two main subunits which are circles and applications of circle. In the first subunit we look at lines and circles, central angle and inscribed angle, and angles formed by two intersecting chords. In the second subunit we look at the applications of circle.

Dear teacher before starting the topics, tell the application of a circle to engage students how much circle is important in your life.

Learning outcomes: At the end of this unit, learners will able to:

- ❖ Have a better understanding of circles
- ❖ Realize the relationship between lines and circles
- ❖ Apply basic facts about central and inscribed angles and angles formed by intersecting chords to compute their measures.
- ❖ Apply real-life situations in solving geometric problems

Suggested Teaching Aids

In addition to the student's textbook and the teacher's guide, you are advised to use colored chalks, ruler, compass, protractor and models of circles.

6.1 CIRCLES

Period allotted: 15 periods

6.1.1 Lines and Circles

Competencies: At the end of this subtopic, students should:

- ❖ Identify the different types of arcs, sectors, segments
- ❖ Describe the concepts "tangent" and "secant" of a circle.

Introduction

It is better to start this subtopic by revising some important parts of a circle. Using a circle, you can explain to the students what is meant by an arc, minor arc, major arc, sector, minor sector, major sector, segment, minor segment, major segment, secant, and tangent to the circle. A given figure where O is the center of the circle, make sure that students indicate parts of a circle. Group work 6.1 helps to revise that they have learnt in grade 6 about circles.

Answer to group work 6.1

a) Center

b) Radius

Answer to Activity 6.1

1) Diameter, chord, arc, etc.
2) Since diameter is twice of the radius, (d = 2r) if diameter = 18cm, then radius = 9cm.
3) They both intersect the circle in two points. But chords are segments and secants are lines.

Answer to Activity 6.2

A line may intersect a circle at no point, one point, or two points.

Answer to Exercise 6.1

1) a) true b) true c) false d) true e) true
2) a) $\overline{AD}$ is a chord b) $\overline{HB}$ is chord c) $\overline{OD}$ is a radius d) $\overleftrightarrow{EG}$ is a tangent to the circle

Answer to Exercise 6.2

1) $\overline{AB}$ is a chord, $\overleftrightarrow{AB}$ is secant, $\overleftrightarrow{CD}$ is a tangent to circle O, region AOP is sector, region ADB is segment, and point P is point of tangency.
2) a) true b) false c) false d) true
3) – line L_1 and circle O do not intersect at all.
   - line L_2 and circle O intersect at A and B. The line L_2 is a secant line.
   - line L_3 and circle O intersect at exactly one-point P. This line is a Tangent
   - Point P is an interior point of circle O
   - Point R is an exterior point of circle O
4) a) In circle O the minor arcs are $\overset{\frown}{NJ}$, $\overset{\frown}{JT}$, $\overset{\frown}{TE}$ and $\overset{\frown}{EN}$
   b) In circle O the major arcs ~~NTE~~, ~~JEN~~, ~~TEJ~~, and ~~EJT~~
   c) ~~TEJ~~ , ~~JET~~ , ~~TNJ~~

6.1.2 Central angle and Inscribed angle

Competencies: At the end of this subtopic, students should:

- ❖ Describe the central angles and inscribed angles
- ❖ Find the measure of central angle or inscribed angle or the intercepted arc based on the given information.

INTRODUCTION

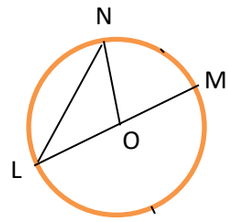
Students are expected to have some background about angles in the previous grades. In this subtopic you will discuss about angles.

You can state and prove theorems on angle and arcs determined by lines (chords) intersecting on a circle or at the center of the circle.

Teaching notes

The main idea of this topic is central angles and angles with vertices on a circle that is inscribed angle. Make sure students relate the measure of these angles with the measure of arcs subtending them and central angles with inscribed angles where the same arc subtending them.

Consider the following figure:



O is the center of the circle

i) $m(\angle MON) = m(\overset{\frown}{MXN})$

ii) $m(\angle MLN) = \frac{1}{2}\, m(\overset{\frown}{MXN})$

Show them, that $m(\angle MON) = 2\, m(\angle MLN)$ from the triangle

It follows that $m(\angle MNL) = \frac{1}{2}\, m(\overset{\frown}{MYL}) = \frac{1}{2}\, m(\overset{\frown}{MXL})$

Therefore, $m(\angle MNL) = \frac{1}{2}(180^0) = 90^0$

Answer to Activity 6.3

1) Inscribed angle

2) a) arc AC

   b) arc AC

Answer to Exercise 6.3

1) a) false b) false c) true d) true e) false f) false

2) a circle measures 360^0

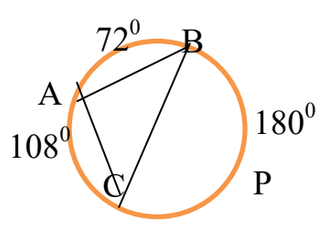
since $3x + 2x + 5x = 360^0$

$10x = 360^0$

$x = 36^0$

Therefore, the measure of the arcs with the ratio 3:2:5: are 108^0, 72^0 and 180^0 respectively.

The figure becomes



$m(\angle CAB) = \frac{1}{2}\, m(CPB)$

$m(\angle CAB) = \frac{1}{2}\,(180^0)$

$= 90^0$

The largest measure of a triangle is 90^0.

3) a) since $\angle BAD \cong \angle DCB$ because they intercept the same arc.

$m(\angle DCB) = 40^0 = m(\angle BAD)$

$\angle ADC \cong \angle CBA$ because they intercept the same arc

$m(\angle ADC) = 42^0 = m(\angle CBA)$

b) $m(\angle ADC) = \frac{1}{2}\, m(\overset{\frown}{ABC})$

$m(\overset{\frown}{ABC}) = 2(120^0) = 240^0$ and

$m(\overset{\frown}{ABC}) + m(\overset{\frown}{ADC}) = 360^0$

$m(\overset{\frown}{ADC}) = 360^0 - 240^0$

$= 120^0$

$m(\angle ABC) = \frac{1}{2}\, m(\overset{\frown}{ADC})$

$= \frac{1}{2}(120^0) = 60^0$

c) $m(\overset{\frown}{AB}) = 2\, m(\angle ACB)$

$= 2(40^0)$

$= 80^0$

$m(\overset{\frown}{AB}) + m(\overset{\frown}{AC}) = 180^0$

$m(\overset{\frown}{AC}) = 180^0 - 80^0$

$= 100^0$

Similarly, $m(\overset{\frown}{AC}) + m(\overset{\frown}{CD}) = 180^0$

$m(\overset{\frown}{CD}) = 180^0 - 100^0$

$= 80^0$

And hence, $m(\angle CBD) = \frac{1}{2}\, m(\overset{\frown}{CD})$

$= \frac{1}{2}(80^0) = 40^0$

Or, $m(\overset{\frown}{AB}) = m(\angle AOB) = 80^0$

and $\angle AOB \cong \angle COD$ …….. vertical opposite angles

then, $m(\angle COD) = 2\, m(\angle CBD)$

$m(\angle CBD) = \frac{1}{2}\, m(\angle COD)$

$= \frac{1}{2}\,(80^0)$

$= 40^0$

4) $m(\overset{\frown}{AB}) = m(\overset{\frown}{BAE}) - m(\overset{\frown}{AE})$

$= 100^0 - 60^0$

$= 40^0$

Now let us find $m(\overset{\frown}{BC})$

$m(\overset{\frown}{BC}) = 180^0 - 60^0 - 40^0$

$= 80^0$

$m(\angle BAC) = \frac{1}{2} m(BC) = 40^0$

$m(DEA) + m(AB) + m(BC) + m(CD) = 360^0$

$m(CD) = 360^0 - 280^0 = 80^0$

$m(\angle CED) = \frac{1}{2} m(CD)$

$= \frac{1}{2}(80^0) = 40^0$

Answer to Group work 6.2

i) It passes through point O, the center of the circle

ii) They are perpendicular

iii) The perpendicular line bisects the chord.

Assessment

You can assess students understanding by following their work in each of the activities, and the class discussion. You can also give them exercise similar to the one given in Exercise 6.2 as class work or homework or assignment. It is also possible to give them a test / quiz which helps assess students.

6.1.3 Angles formed by two intersecting chords (4 periods)

Competencies: At the end of this topic, students should:

- Describe the angle formed by two intersecting chords
- Solve problems related to angle formed by two intersecting chords inside a circle.

INTRODUCTION

In the previous lesson, give descriptions of central angel and inscribed angle together with what a measure mean. Next discuss angles formed by two intersecting chords. And this is stated in theorem 6.3 which is the measure of an angle formed by two chords intersecting inside a circle is one-half of the sum of the measure of the arcs subtending the angle and its vertical opposite angle.

Answer to Activity 6.4

$m(\angle APD) = \frac{1}{2}(m(AD) + m(BC))$

$= \frac{1}{2}(72^0 + 48^0)$

$= \frac{1}{2}(120^0)$

$= 60^0$

Answer to Exercise 6.4

1) a) true b) false c) true

2) a) $m(\angle APB) = 180^0 - 110^0 = 70^0$

$m(\angle BPC) = \frac{1}{2}(m(BC) + m(AD))$

$m(BC) + m(AD) = 2\, m(\angle BPC)$

$= 2\,(70^0)$

$= 140^0$

b) first find the $m(\angle ABD)$

$m(\angle BAC) + m(\angle APB) + m(\angle ABD) = 180^0$ …… angle sum theorem

$m(\angle ABD) = 180^0 - 150^0 = 30^0$ and

$\angle ABD \cong \angle ACD$ ……..they intercept the same arc

Therefore, $m(\angle ACD) = 30^0$

3) $m(AD) + m(DC) + m(CB) + m(BA) = 360^0$

$m(AD) = 2\, m(\angle ABD) = 2(40^0) = 80^0$

$m(DC) = 2\, m(\angle CBD) = 2(45^0) = 90^0$

$m(AB) = 90^0$

$m(CB) = 360^0 - 260^0$

$= 100^0$

Therefore, $m(\angle APD) = \frac{1}{2}(m(AD) + m(BC))$

$= \frac{1}{2}(80^0 + 100^0)$

$= 90^0$

6.2 Application of circle

Allotted period: 2 periods

Competency: At the end of this topic, students should:

- ❖ Solve the application problems

INTRODUCTION

In this section, we will discuss the application of circle. Circles are present in real life, both in the natural world and in man-made creation Such as nature surrounding human's technology, architecture, and mappings.

Dear teacher please guide students to discuss the application of circle in your daily life. Some examples of application of circles are; satellite's orbit around the earth, Mushrooms with domed caps, Camera lenses, Coins, Cups, Rings, Wheel, roundabout, a sniper target grid, etc.

Assessment

Dear teacher, at this time you have analyze the understanding of your students. Please assess your students' based on the following concepts on lines and angles in a circle.

- ✓ Give different problems to describe the different parts of a circle
- ✓ Give different problems to identify the relationship between lines and circles
- ✓ Give different exercises to describe the perpendicular bisector of a chord
- ✓ Give different exercises to describe central and inscribed angles
- ✓ Give different exercises to describe the relationship of central and inscribed angles with an arc intercepted them
- ✓ Give different exercises to calculate the measure of central and inscribed angles
- ✓ Give different exercises to describe an angle formed by two intersecting chords
- ✓ Give different problems to analyze the application of circle.

Make a short review using the summary of the unit and give some questions of the review exercise on unit 6 to be done in class. Select also some for an assignment.

Answer to Review exercise

1) a) false b) true c) false d) true e) true f) true

2) i) a) $\angle ACD \cong \angle ABD$ …….they intercept the same arc

$m(\angle ACD) = m(\angle ABD) = 45^0$

b) $m(\angle ACD) = \frac{1}{2} m(BC)$

$= \frac{1}{2}(100^0)$

$= 50^0$

c) first find $m(AD)$,

$m(AD) = 2\, m(\angle ABD) = 90^0$

$m(\angle BPC) = \frac{1}{2}(m(BC) + m(AD))$

$= \frac{1}{2}(50^0 + 90^0)$

$= 70^0$

ii) No, because $m(\angle BPC) \neq m(AD)$ that is $\angle BPC$ is no central angle.

iii) $m(\angle ACD) = m(\angle ABD) = 45^0$

$m(\angle BDC) = m(\angle BAC) = 50^0$

$m(\angle DPC) = 180^0 - 95^0 = 85^0$

Therefore, the triangle is acute angle triangle.

3) $m(\angle POS) = 95^0$, then $m(\angle QOR) = 95^0$

a) $m(\angle QSR) = \frac{1}{2} m(\angle QOR)$

$= \frac{1}{2}(95^0)$

$= 47.5^0$

b) $m(\angle PQS) = \frac{1}{2} m(\angle POS)$

$= \frac{1}{2}(95^0)$

$= 47.5^0$

c) $m(\angle QPR) = m(\angle QSR) = 47.5^0$

4) Fist find $m(DE)$,

$m(\angle BFC) = \frac{1}{2}(m(BC) + m(DE))$ but

$m(BC) = 140^0 - 50^0 = 9^0$

$m(DE) = 2\, m(\angle BFC) - m(BC)$

$= 2(80^0) - 90^0$

$= 70^0$

$m(\angle DAE) = \frac{1}{2} m(DE)$

$= \frac{1}{2}(70^0)$

$= 35^0$

5) $m(\angle BCD) = 90^0$

$m(BAD) = 2\, m(\angle BCD) = 2(90^0) = 180^0$

Since $\overline{BD}$ is a diameter.

a) $m(\angle ACB) = m(\angle ACD) = 61^0$

b) First, find $m(CD)$

$m(BC) = 2\, m(\angle BAC) = 2(49^0) = 58^0$

$m(CD) = 180^0 - m(BC)$

$= 180^0 - 58^0$

$= 122^0$ and

$m(ADC) = m(CD) + m(AD)$

$= 61^0 + 58^0$

$= 119^0$

$m(\angle ABC) = \frac{1}{2} m(ADC)$

$= \frac{1}{2}(119^0)$

$= 59.5^0$

c) $m(\angle CAD) = \frac{1}{2} m(CD)$

$= \frac{1}{2}(122^0)$

$= 61^0$

d) $m(\angle BEC) = \frac{1}{2}(m(BC) + m(AD))$

$= \frac{1}{2}(58^0 + 58^0)$

$= 58^0$

6) $m(TP) = 2\, m(\angle PBT)$

$= 2(70^0)$

$= 140^0$

Since, $\overline{PQ}$ is a diameter

$m(TQ) = 180^0 - m(TP)$

$= 180^0 - 140^0$

$= 40^0$

$m(\angle TPQ) = \frac{1}{2}\, m(TQ)$

$= \frac{1}{2}(40^0)$

$= 20^0$

7) $m(\angle AOC) = m(AC) = 132^0$

$m(\angle COD) = 180^0 - 132^0 = 48^0$

$m(\angle CDA) = \frac{1}{2}\, m(AC)$

$= \frac{1}{2}(132^0)$

$= 66^0$

a) $m(\angle CDO) + m(\angle C0D) + m(\angle OCD) = 180^0$…….angle sum theorem

$m(\angle OCD) = 180^0 - 114^0$

$= 66^0$

b) $m(CD) = m(\angle COD) = 48^0$

8) $m(\angle ACB) = \frac{1}{2}\, m(APB)$

$7x + 16 = 18x - 32$

$14x + 32 = 18x - 32$

$4x = 64$

$x = 16$

Therefore, the measure of the inscribed angle is $7x + 16 = 128^0$

UNIT 7

Solid Figures and Measurement

Total Allotted period: 20 periods

Learning Outcomes: At the end of this unit, learners will able to:

- ❖ Identify parts of solid figures
- ❖ Find the surface area of solid figures
- ❖ Find the volume of solid figures
- ❖ Solve applications of solid figures and measurements

Introduction

There are many applications for solid figures in the real world. In this unit, you will apply the formulas for the surface area and volumes of solids to solve real-world problems. Formulas are derived in many ways, and you will learn to construct nets of solid figures to derive the formula easily. We will derive as well as compare the volumes of different solids and apply the formulas for the surface area of prisms and cylinders to solve real- world problems. In this unit you will learn description of solid figures, name of solid figures according to their properties, surface area of solid figures, and volume of solid figures and solve problem in a real-life applications of solid figures.

Suggested Teaching Aids

You can encourage your students work to draw the net of solid figures such as prisms and cylinders folding papers. You can use the following materials present clearly the lesson

- ✓ Computer and projector for video lesson
- ✓ Rectangular prism (book)
- ✓ Cube (Rubik's Cube)
- ✓ Cone (Birth day hat)
- ✓ Pyramid (paperweight or make a pyramid with Legos)
- ✓ Sphere (dodge ball or globe)
- ✓ Cylinder (can)
- ✓ Paper
- ✓ Pencils
- ✓ Pictures of real-life objects representing solid figures

7.1 Solid figures

Period Allotted: 4 periods

Competencies: At the end of this section, students should:

- ❖ Identify parts of a prisms, cylinders, pyramid and cones
- ❖ Name different types of prisms, cylinders, pyramid and cones based on their bases.

Solid figures are three-dimensional objects. What this means is that solid figures have a width, a length, and a height. For instance, computer, laptop and phonetic have a width, a length, and a height.

In mathematics, there are many solid figures. Most of them are prisms, Cylinders, pyramids and Cones.

7.1.1: Prisms and Cylinders

We encounter prisms and cylinders everywhere; most boxes and most rooms are rectangular prisms, most cans are cylinders. When we find out how much cardboard there is in the box, when we need the area of the walls to paint in a room, or when we need to find how much tin is needed to make a can, we are finding the **surface area of prisms** and **cylinders**. When we find out how much milk is in the container, how much soup is in the can, and how much chocolate is in the packet, we are finding the **volume of prisms** and **cylinder**?

Since the students have well established background on geometric figures. You can stat the lesson by asking the following choice questions.

1. How many straight lines can be drawn through two given lines?
   A) None
   B) Only one
   C) Two
   D) Three
2. What is the minimum number of lines required to make a closed figure?
   A) One
   B) Two
   C) Three
   D) Four
3. How many dimension does a surface has?
   A) One
   B) Two

C) Three

D) Four

4. A solid has how many dimensions?

   A) One

   B) Two

   C) Three

   D) Four

5. What do you call a figure formed by two straight lines having a common point?

   A) Angle

   B) Triangle

   C) Rhombus

   D) Kite

7. How many lines can pass through one point?

   A) One

   B) Two

   C) Three

   D) Four

8. Which of the following are boundaries of a surface?

   A) Lines

   B) Curves

   C) Surfaces

   D) Points

The purpose of group work 7.1 is to name and identify solid figures.

Answer for Group work 7.1

1. Cone, Prism, Pyramid, Cube, and Cylinder
2. a) Model examples of prisms are laptops, cartons box, rectangular box, ice cube, barns, camping tents, text books, fish tanks, cargo containers and rooms.

   b) Model example of cylinder is pipes, cold drink cans, water tanks, battery, gas cylinder, candle, test tube and beakers.
3. A cylinder is a three dimensional solid that holds two parallel bases joined by a curved surface. These bases are a circular disk in shapes

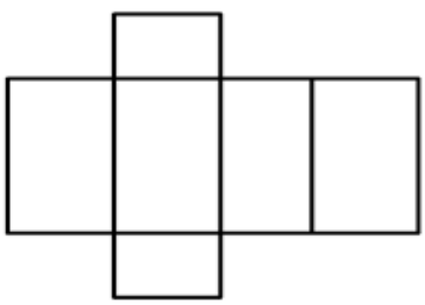
   A prism is athree dimensional figure in which two of the faces, called the bases of the prism, are congruent polygons in parallel plane.

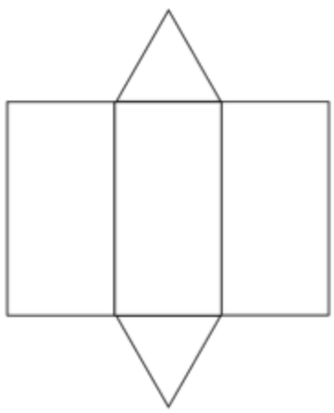
4. A prism has a solid shape consisting of two identical ends (such as triangle, square, rectangle, etc.), flat faces or surfaces and uniform cross-section across its length such as edges, faces and vertex

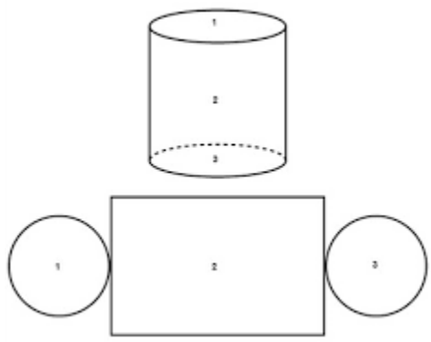
   A cylinder parts are the bases are two circles, perpendicular to the axis. They are the covers that close the cylinder. The height is the distance between the bases. The radius (r) is the length from the axis to the extreme of the cylinder and corresponds to the radius of the base.
5. There are a total of 5 faces, 9 edges, and 6 vertices in a triangular prism.

There are a total of 3 faces, 2 curved edges, and no vertices in circular cylinder.

6. A cube is a rectangular prism with all sides equal, and therefore all faces of equal area. A rectangular prism will have 4 faces of equal area and 2 faces of equal area. So a cube is a rectangular prism, but not all rectangular prisms are cubes.
7. Rectangular prism, Triangular prism and Hexagonal prism respectively
8.







Answer for Exercise 7.1

1. a), True b), True c), True d), True e), False f), True g), True
2. The difference between prisms and cylinders are

   a) A cylinder is a geometrical figure of revolution while a prism is not.

b) A cylinder consists of 2 flat ends and a curved surface while a prism contains two polygons for the two ends and the remaining are plain rectangular faces.

c) A cylinder does not have any diagonals while a prism contains many.

d) A cylinder consists of only one shape while a prism has many shapes depending on the shape of the two ends.

e) A cylinder has no vertices while a prism has various vertices.

f) A cylinder contains 2 curved edges while a prism has no curved edge

The Similarities of prisms and cylinders are

A cylinder and a Prism are similar if you imagine properly. Imagine a prism with regular polygons for bases, as you increase the number of sides, the solid gets to look exactly as a cylinder. Hence, Cylinder is a prism with infinite number of faces. Both of the prisms and cylinders have their base and top as congruent faces and parallel to each other. Also, a prism becomes a cylinder as the number of sides of its base becomes larger and larger.

3. a) Circular Cylinder b) Rectangular prism
4. Please teacher using draw the following two solids by using ruler
5. There are a total of 6 faces, 12 edges, and 8 vertices in a triangular prism.

There are a total of 3 faces, 2 curved edges, and no vertices in circular cylinder.

Assessment

You should assess each student's work continuously over the whole periods and discuss it with the following points

- Ask questions and let some students to identify parts prisms and cylinders by net of prisms and cylinders
  a) Using Geogerba Software
  b) Using manually.
- Provide constrictive feedback for learners to improve their learning.
- Give project on the parts of prisms and cylinders and the teacher checks the performance of students.

For slow learner students

1. Choose the correct answer

a) To find the total surface area of rectangular prism, you must calculate and add the areas of

A, 2 faces B, 3 faces C, 4 faces D, 6 faces

b) A 3-D object with two parallel and congruent circular bases is

A, Cylinder B, Rectangular prism C, Cone D, Triangular prism

c) What 3- D solid does this net create?

A, Triangular prism

B, Rectangular prism

C, Triangular pyramid

D, Square prism

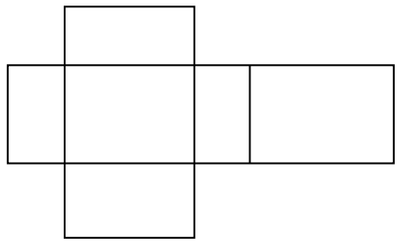
d) Which 3-D object can be created by folding this net?

A, Triangular prism

B, Rectangular prism

C, Triangular pyramid

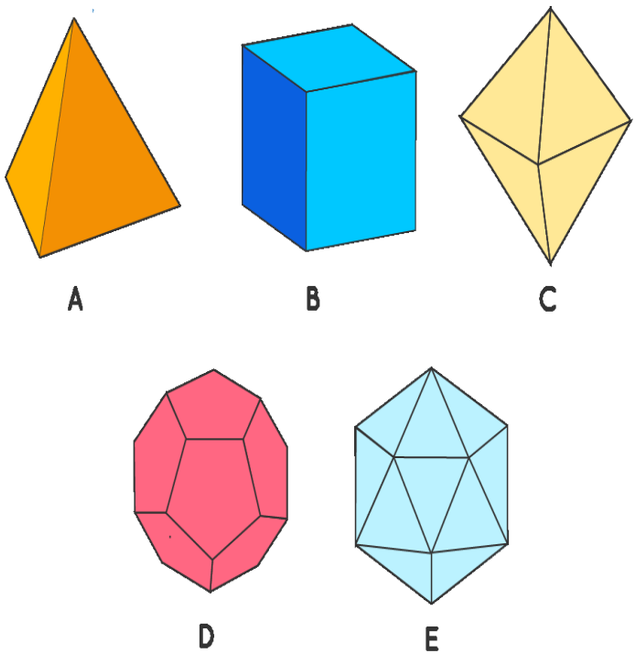
D, Cylinder



e) Which combination of views is the minimum required to describe a 3-D objects?

A, Front only B, Front, side and top C, Side and front D, Side and top

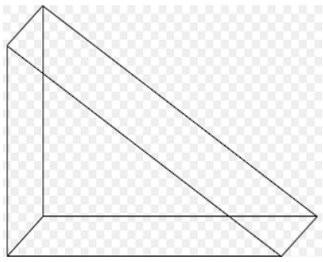
f) Which of the following solid figures has the maximum number of faces?



For fast learner students

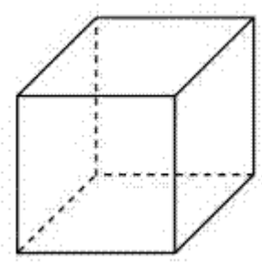
2. Name and describe each of the following solid figures

a)



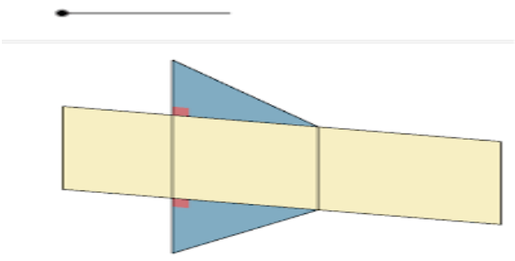
b)

c)

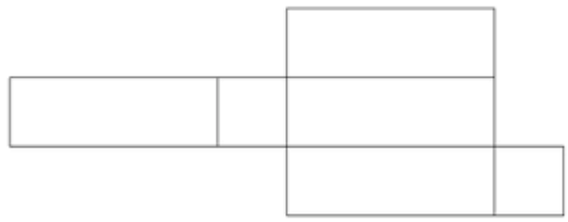


3. Folding the following net figures and name

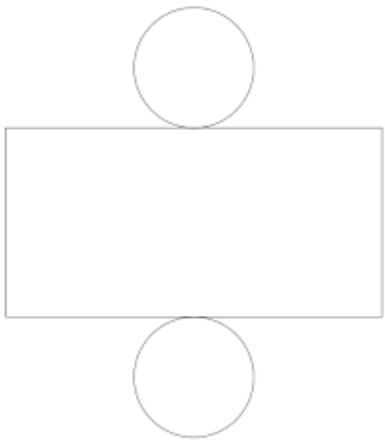
a)



b)



c)



4. Find number of edges, faces and vertices of the following solid figures
   a) Rectangular prism
   b) Triangular prism
   c) Cylinder
   d) Square prism

7.1.2. Pyramids and Cones

The main focus of this sub-unit is to identify and name the edge, vertex, altitude and face of pyramids and cones.

Answer for activity 7.1

1. A cone is a solid three-dimensional figure with a circular base and one vertex.
   A pyramid is a three-dimensional object with a base that is a polygon and triangular faces that meet at one vertex. ... Base Edge The base edge is the edge between the base and the lateral faces of a prism.
2. Model example of Pyramids is the Great Pyramid of Giza. the roof of a house. Some free-standing cheese graters. glass Lourve Pyramid.
   Model examples of cones in daily life: Ice cream cone, Funnel, Christmas tree and Birth day hat.
3. a) Vertex of pyramid V and Vertex of Cone V
   b) ABCD

c) ΔVDA, ΔVDC, ΔVAB and ΔVBC

d) $\overline{VO}$

For depends the concepts of definition of pyramids and cones, students should practice using Exercise 7.2

Answer for Exercise 7.2

1. a) False b) True c) False d) True e) True
2. The differences between Pyramid and cone are
   a) A pyramid is the cone with a polygonal base and A cone has non polygonal bases(curve)
   b) A cone has only 1 curved edge whereas and a pyramid has a minimum of 6(for a triangular pyramid).
   c) The area of the pyramid is depend on base of the polygon

   Similarities of pyramids and cones are

a) Both have a base which extends into a single vertex.
b) A cone and a pyramid both have an apex.
c) The cone I am referring to in the comparison below to a pyramid is not the mathematical cone, but a truncated half cone)
d) Both cone and pyramid have a bottom surface (base) and a height
e) Both have a volume of 1/3 the area of the base times the height

3. No, it is not possible to have rectangular lateral faces because a pyramid is formed with a polygonal base and the lateral sides are triangles
4. Please teacher sketch the solid figures by using teaching materials to clear ideas for students question a and b in students text book.

7.2. Surface Area and Volume of Solid Figures

Total Allotted period :(13periods)

7.2.1. Surface area of Prisms and Cylinders

Competency: At the end of this section, students should:

- Find the surface area of prisms and cylinders

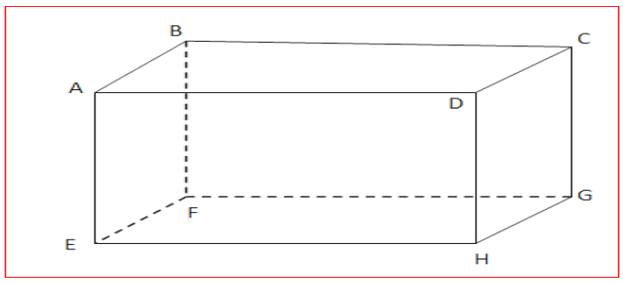
In this sub-section you will become more acquainted with these familiar geometric figures and you will learn how to compute their surface area in a systematic way.

In geometry, a net is a 2-dimensional shape that can be folded to form a 3-dimensional shape or a solid. Or a net is a drawing made when the surface of a 3-dimensional figure is

laid out flat showing each face and edge of the figure in 2-dimension. Nets are helpful when we need to find the surface area of the solids.

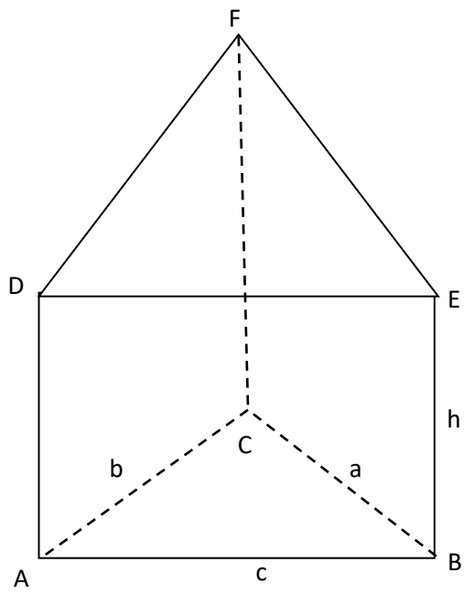
Surface area is a very fundamental property of 3-dimensional shapes. Often times in geometry we will be asked to find the surface area of a shape. The more simple shapes can be solved by using a general formula. More complex shapes will require us to apply our knowledge of one or several 2-dimensional shapes. If you can see a triangular prism when unfolded consists of two triangles and three rectangles. The triangles are the bases of the prism and the rectangles are the lateral faces. The surface area is the sum of the areas of the five shapes. So the surface area of a 3-dimensional object is the measure of the total area of all its faces. This means that one way to find the surface area of a solid is to find the area of its net.

Answer for Group work 7.2



Answer Group work 7.3

1. Consider the following triangular prism



Derive a formula of surface area of triangular prism using 3D shape

It has 5 faces, two bases and 3 faces

Lateral Surface Area = the sum of the area of the three lateral faces.

A_L= a(ABED) + a(BCFE) + a(ACFD)

$$= ch + ah + bh$$

$$= h(c + b + a)$$

$$= ph$$

A_T = A_L + areas of the two bases

= A_L + a(ΔABC) + a(ΔDEF)

= A_L + ab

Answer for Exercise 7.3

1. a) True, b) True, c) False d) False e) True f) True
2. $A_L = ph$

   = (3cm + 6cm +7cm)

   (16cm)(10cm) = 160 cm^2.
3. $A_L = ph$

   = 100cm×5cm

   = 500 cm^2.

   A_T= AL + 2AB

   = 500 cm^2 + 2(50cm^2)

   = $600cm^2$
4. $A_L = 2\pi rh$

$$88\pi cm^2 = 2\pi r \times 8cm$$

$$5.5cm = r$$

But $d = 2r = 11cm$

Therefore, the diameter of circular cylinder is 11cm.

5. Let $r_1 = 3cm$ and $r_2 = 12cm$ therefore the ratios of the two radius is 1: 4 or 4: 1

Assessment

Dear teacher, at the end of this sub, unit apart from exercise 7.3 you can

- ❖ Ask questions and let some students how to derive the surface area formula of cylinder by net of cylinder.
  a) Using Geogerba Software
  b) Using manually.
- ❖ Provide constrictive feedback for learners to improve their learning.
- ❖ Give project on the surface area of cylinder and the teacher checks the performance of students.
- ❖ Give assignments, quiz or test to assess their level of understanding.

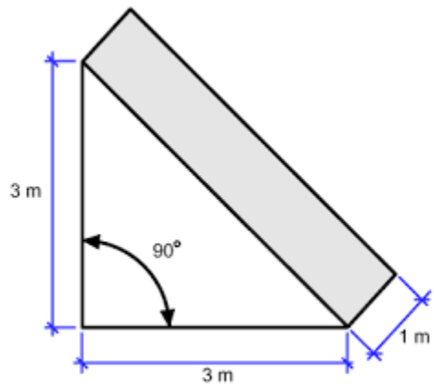
Finally, for slow learner students and fast learner students, you can give the following additional exercise

For slow learner students

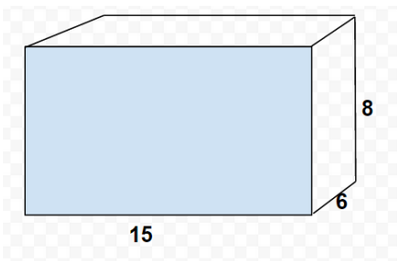
1. Write true if the statement is correct and false if it is not
   a) Surface area is the amount of area of the surface of a 3D figures
   b) The base of a triangular prism is a rectangle
   c) Lateral surface area does include bases
   d) The formula to find lateral surface area of a cylinder is $S = 2\pi rh$
   e) Surface area of a prism is always less than the volume of a prism
2. A right circular cylinder has a base with a diameter 7cm and height 10cm. Find total surface area of a cylinder
3. Find the total surface area of a triangular prism whose bases are equilateral triangles of side 4cm and height is 5cm

For fast learner students

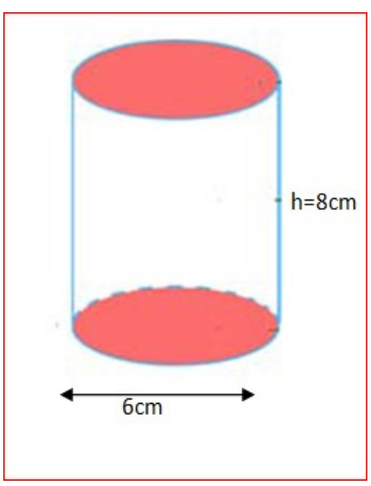
4. A square prism's total surface area is $30cm^2$ and the length of the side of square base is 1cm. find the height of the prism's
5. The lateral surface area of a right circular cylinder is 120cm^2 and the circumference of the bases is 12cm. Find the altitude of the cylinder.
6. Find total surface area of the following solid figures
   a)



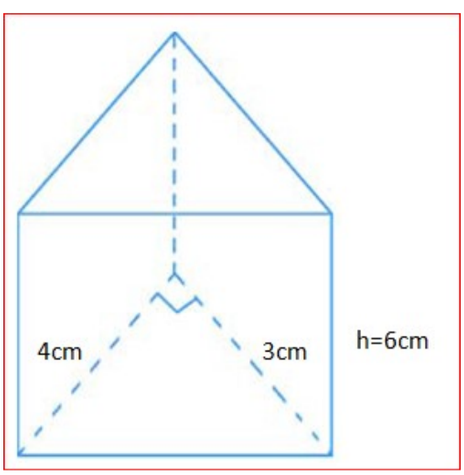
b)



c)



d)



7.2.2. **Volume of Prisms and Cylinders**

Competency: At the end of this sub- unit, students should:

- ❖ Find the volume of prisms and cylinders.

The **volume of a solid** is the *amount of space inside the object.* It's how much water fits inside a bathtub, how much sand fills a bucket, or how much soda your friend can chug and hold in his stomach.

Volume of a rectangular prism = (area of base) (height)

Let's look at another type of prism: a cube. The length, width, and height of a cube are all equal. It's a close relative to the square. A piece of origami paper (think thin, colorful, easy to fold) is a square, and a stack of origami paper is a cube if it's as high as the paper is wide (or long).

If each piece of origami paper is 5 inches long and 5 inches wide, the area of the piece of paper on the base of the stack is 25 inches^2. In other words, the area of the base is 25 in^2. If the stack is 5 inches high, then it has a volume of 25 in^2 × 5 inches of paper, or 125 in^3.

If we slide the top of the stack over, but the base stays where it is, we can make a leaning tower of Pisa, or an oblique prism. While the shape of the prism has changed a bit, the volume of the prism has not. The same amount of paper is still there. The volume of an oblique prism is also (area of base)(height), but the height has to be perpendicular to the base. In the case of the mysterious leaning tower of origami, the height is the distance from the top piece of paper straight down to the surface it sits on. *Volume is always measured in cubic units (units^3). Similarly*

Volume of a Prism or Cylinder = *area of the Base x height*

Answer for activity 7.2

1. The volume of a solid is the number of cubic units contained in its interior. That is, the amount of space it occupies. Volume is measured in cubic units such as m^3, cm^3 etc.

2. Volume is expressed as the product of the area of the base and the height. The two solid figures that will have the same volume are the solid in A and D. The rectangular solid will have a volume of 72 cm^3 and the oblique solid will, as well, have a volume of 72 cm^3

   Yes because Volume is determined by the product of base area and height

Therefore if the volume of two solid have the same base and height then They are equal.

3. V = $A_B \times h$.

   V = $48cm^2 \times 18cm$

   V = $864cm^3$

4. V = $\pi r^2 h$

$$784\pi cm^3 = \pi r^2 \times 16cm$$

$$r^2 = 49cm^2$$

$$r = 7cm$$

Answer for Group work 7.3

Let r_1 and r_2 be smallest and largest radius respectively

Let h_1 and h_2 be smallest and largest height respectively

Let c_1 and c_2 be smallest and largest circumferences respectively

$$c_1 = 2\pi r_1$$

$$12cm = 2\pi r_1$$

$$r_1 = \frac{6}{\pi} cm$$

$$c_2 = 2\pi r_2$$

$$r_2 = \frac{28}{2\pi} cm$$

$$r_2 = \frac{14}{\pi} cm$$

$$V_1 = \pi {r_1}^2 h_2$$

$$V_1 = \pi {r_1}^2 h_2$$

$$V_1 = \pi \times (\frac{6}{\pi} cm)^2 \times 12cm$$

$V_1 = \frac{432}{\pi} cm^3$ And

$$V_2 = \pi {r_2}^2 h_2$$

$$V_2 = \pi {r_2}^2 h_2$$

$$V_2 = \pi \times (\frac{14}{\pi} cm)^2 \times 28cm$$

$$V_2 = \frac{5488}{\pi} cm^3$$

Therefore, V_2 is greater than V_1 because the height and radius of V_2 is the greater.

Answer for Exercise 7.4

1. a) True, b) True, c) True
2. V = $A_B \times h$.

$$204cm^3 = A_b \times 17cm$$

$$A_b = 12cm$$

3. $V = L \times W \times h$

$$V = 8cm \times 5m \times 16cm$$

$$V = 640cm^3$$

4. $V = \pi r^2 h$ but $d = 2r \Longrightarrow r = \frac{12}{2}cm \Longrightarrow r = 6cm$

$$V = \pi(6cm)^2 \times 20cm$$

$$V = 720\pi cm^2$$

5. The picture is not drawled

Assessments

You should asses each student's work continuously over the whole period and discuss it the following questions

- Ask questions and let some students how to derive the volume of prisms and cylinder by net of prisms and cylinder.
  a) Using Geogerba Software

b) Using manually.

- Provide constrictive feedback for learners to improve their learning.
- Give activity on the volume of prisms and cylinder and the teacher checks the performance of students

Finally, for slow learner students and fast learner students, you can give the following additional exercise

For slow learner students

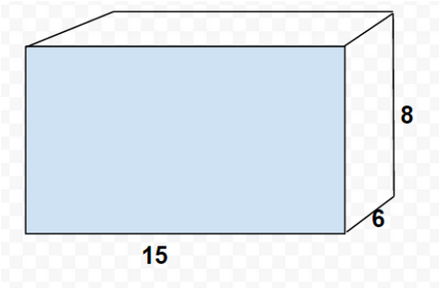
1. Write true if the statement is correct and false if it is not
   a) If the height of a cylinder doubles, then the volume of the cylinder increases by 8times
   b) If the surface area of two cylinder is the same, then the cylinder have the same volume
   c) Volume is a 3-dimensional figure

d) If the edges of a rectangular prism all double, then the volume of the rectangular prism increases by 4times

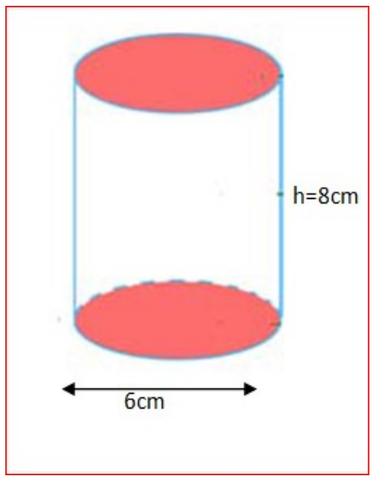
2. Find volume and surface area of a cylinder if the diameter is 14cm and the height is 4cm.
3. A rectangular prism has length 9cm, width 4cm and height 6cm find
   a) Volume of the rectangular prism
   b) Surface area of a rectangular prism
4. The volume of prism of a triangular prism is 204cm^3. If its height is 17cm. find base area?

For fast learner students

5. Write true if the statement is correct and false if it is not
   a) The surface area of a cuboid 7cm long, 5cm wide is 214cm^3.Its height is 3cm
   b) The radius of a cylindrical tank is doubled while the lateral surface area remains unchanged. The height will be halved
   c) When the volume and surface area of a square prism are numerically equal. Then its edge is equal to 6
   d) The volume of water tank is measured in square units
   e) If radius and height of a cylinder are doubled, then its volume becomes eight times
6. The volume of a cylinder is 660cm^3. Find its height if its radius is 5cm.
7. Find the volume and surface area of a cube whose edge is 10cm.
8. Find the volume of the following solid figures
   a)



   b)



7.3 Applications on Solid figures and Measurements

Allotted period: 3periods

Competency: At the end of this sub- unit, students should:

- Solve real-life problems Applications on Solid figures and Measurements

Introduction

Solid figures are fat not flat. Find a cone in a birthday hat, A square prism in a building, Cube in the dice roll, and a cylinder in shiny flag pole

Moving your fingers along geometric shapes will help you understand the concept of faces, edges, and vertices.

Solids or three-dimensional objects have 3 dimensions, namely length, width, and height.

Solid shapes have faces, edges, and vertices.

Learning about solid shapes will help us in our day-to-day life as most of our activities revolve around and depend on them.

Answer for Exercise 7.4

1. $V = \pi r^2 h$ but $r = 6cm$ and $h = 11cm$

$$V = \pi(6cm)^2 \times 11cm$$

$$V = 396\pi cm^2$$

$A_T = AL + 2A_b$ But $AL = 2\pi rh = 2\pi \times 6cm \times 11cm = 132\pi cm^2$

$$A_b = \pi r^2$$

$$A_b = \pi(6cm)^2$$

$$A_b = 36\pi cm^2$$

$$A_T = 132\pi cm^2 + 2(36\pi cm^2)$$

$$A_T = 204\pi cm^2$$

2. $A_T = AL + 2A_b$ But $AL = 2\pi rh = 2\pi \times 4cm \times 10cm = 80\pi cm^2$

$$A_b = \pi r^2$$

$$A_b = \pi(4cm)^2$$

$$A_b = 16\pi cm^2$$

$$A_T = 80\pi cm^2 + 2(16\pi cm^2)$$

$$A_T = 112\pi cm^2$$

$V = \pi r^2 h$ But $r = 8cm$ and $h = 20cm$

$$V = \pi(8cm)^2 \times 20cm$$

$$V = 1280\pi cm^2$$

3. $V_A = \pi r^2 h$ but $r = 5cm$ and $h = 14cm$

$$V_A = \pi(5cm)^2 \times 14cm$$

$V_A = 350\pi cm^2 = 1099cm^3$ And

$$V_B = l \times w \times h$$

$$V_B = 12cm \times 13cm \times 5cm$$

$$V_B = 780cm^3$$

Therefore, a company stores food A is accommodating more food than food B.

Answer for Review Exercise unit 7

1. a) True, b) True, c) True
2. $V = A_b h$

$$204cm^2 = A_b \times 17cm$$

$$A_b = 12cm$$

3. $V = l \times w \times h$

$$V = 8cm \times 5cm \times 16cm$$

$$V = 640cm^3$$

4. $s^2 = 81cm^2$

$$s = \sqrt{81cm^2}$$

$$s = 9cm$$

AL = ph

$144cm^2$ = (9cm+9cm+9cm+9cm)$\times$ h

$$144cm^2 = 36hcm$$

$$h = 4\text{cm}$$

5. a) $A_L = ph$

   144cm^2 = (9cm+9cm+9cm+9cm)× 15cm

   A_L =90cm×15cm

$$A_L = 1350\text{cm}^2$$

   b) 9cm, 40cm and 41cm

   c) $A_T = A_L + 2A_B$ But $A_B = \frac{1}{2} \times 9\text{cm} \times 40\text{cm}$

$$A_B = 180\text{cm}^2$$

$$A_T = 1350\text{cm}^2 + 2(180\text{cm}^2)$$

$$A_T = 1710\text{cm}^2$$

6. $A_T = 2\pi r(h + r)$

$$884\pi\text{cm}^2 = 2\pi \times 2\text{cm}(h + 2\text{cm})$$

$$884\pi\text{cm}^2 = 4\pi\text{cm}(h + 2\text{cm})$$

$$884\pi\text{cm}^2 = 4\pi 2\text{cm}h + 8\pi\text{cm}^2$$

$$884\pi\text{cm}^2 - 8\pi\text{cm}^2 = 4\pi\text{cm}h$$

$$h = 219\text{cm}$$

7. r be radius and h be the height of cylinder

$$h + r = 9\text{cm}$$

$$A_T = 81\text{cm}^2$$

$$A_T = 2\pi r(h + r)$$

$$81\text{cm}^2 = 2\pi r(9\text{cm})$$

$$r = 4.5\text{cm}$$

8. $3x + 2x + x = 138\text{cm}^2$

$$6x = 138\text{cm}^2$$

$$x = 23\text{cm}$$

   $l = 3x = 69\text{cm}$, $w = 2x = 46\text{cm}$ and $x = 23\text{cm}$

$$AL = ph = 2(l + w) \times h = 2(69\text{cm} + 46\text{cm}) \times 23\text{cm}$$

   $AL = 5290\text{cm}^2$ And $A_B = l \times w = 69\text{cm} \times 46\text{cm} = 3174\text{cm}^2$

$$A_T = AL + 2(A_B)$$

$$A_T = 5290\text{cm}^2 + 2(3174\text{cm}^2)$$

$$A_T = 5290\text{cm}^2 + 6348\text{cm}^2$$

$$A_T = 11638\text{cm}^2$$

9. $V_L = l \times w \times h$

$$V_L = 16\text{cm} \times 11\text{cm} \times 9\text{cm}$$

$V_L = 1584cm^3$ and

$$V_s = l \times w \times h$$

$$V_s = 2cm \times 2cm \times 2cm$$

$$V_s = 8cm^3$$

Therefore, number of small boxes fit into the bigger box is 198

10. $V = \pi r^2 h$

$$252cm^3 = \pi(4cm)^2 \times h$$

$$252cm^3 = 16\pi cm^2 h$$

$$h \approx 5cm$$

11. $V = A_b h$

$$116\pi cm^2 = 124cm \times h$$

$$h \approx 3cm$$

12. $A_T = 6s^2$

$$A_T = 6(13cm)^2$$

$$A_T = 1014cm^2$$

$$V = l^3$$

$$V = 13cm \times 13cm \times 13cm$$

$$V = 2197cm^3$$

UNIT 8

Introduction to Probability

Total Allotted period: 14 periods

Learning Outcomes: At the end of this unit, learners will able to:

- Understand the concept of probability
- Find event, sample space and probability of simple events.
- Apply problems of real-life situations in solving the probabilities

Introduction

It is quite common phenomena to relate success or failure in one thing with luck or chance. Trying to determine how likely something can occur is useful in applications. For such purpose, discussing probability is essential which hill help represent such concepts as chance or luck mathematically. In this unit, students will discuss introductory concepts of probability starting from the definition of basic terms and real- life applications.

Suggested teaching aids

You know that students learn in a variety of different ways. Some are visually oriented and more inclined to acquire information from photographs or videos. Others do best when they hear instructions rather than read them. Teachers use teaching aids to provide these different ways of learning. Therefore, it is recommended that you might use models, fair dice, different coins and slip of paper for probability.

8.1. The concepts of probability

Competency: At the end of this sub-unit, students should:

Describe the concepts of probability.

This sub-unit begins with discussing the definition of basic terms of probability, which are experiment, sample space, event, certain event and impossible outcomes. Finally, dear teacher should make sure that students are using the activities, group work, and challenge problem of the textbook. To stabilize the lesson taught in each sub- unit

Teaching notes

Under each sub-topic, a hint given how to continue each sub-topic but your creativity is very crucial. The purpose of the teaching notes is to provide the teacher information to use activities, opening problems and group works to motivate and guide students rather than lecturing.

Probability is a number that reflects the chance or likelihood that a particular event will occur. Probability can be expressed as proportions that range from 0 to 1, and they can also be expressed as percentages ranging from 0% to 100%. A probability 0 indicates that there is no chance that a particular event will occur; where as probability of 1 indicates that an event is certain to occur

Established, more emphasis will be given to the different methods of finding the

Probability of an event. While discussing an experiment, it is essential for the students to realize that the outcome of an experiment (if conducted randomly) is not known. Thus, the probability of an event under such a condition will be determined only after the experiment is conducted at full scale. As an implicit concern, however, after the students are made clear with the notions of experiment, sample space and event, the notion of probability of an event established under a random experiment with equally likely outcomes can be known before conducting the experiment or even without conducting the experiment. In order to assist students reach such an understanding, it is recommended to let them do Group Work 8.1 first.

Answer for Group work 8.1

In each of the first three experiments, the result or outcome is certain, and known in advance. That is, in experiment (a),the ball is certain to touch the earth and in (b) the sugar will certainly dissolve in milk and in (c) the petrol is sure to burn. But in the experiments d – e the results are uncertain. For example, when a coin is tossed everyone knows that there are two possible outcomes namely head and tail but no one could say with certainly which of the two possible outcomes will be obtained. In all, such experiments, that there is an element of chance, called probability which express the element of chance numerically. The occurrence or non-occurrence of things is studied in mathematics by the theory of probability. The theory of probability was introduced to give quantification to the possibility of certain outcome of the experiment in the face of uncertainty. Governments, scientists, insurance company, use probability

After Group work 8.1, you can formally write the definition of an experiment, a sample space and an event as stated in the student textbook. One important point in the discussion made so far is that the proportions or the probabilities could be identified only after the experiment is conducted. Once an experiment is performed, it will be thus possible to determine the probability. For this purpose, let each student do Exercise 8.1

Answer for Exercise 8.1

1. a) An activity involving chance in which results are observed is called an experiment.
   b) The set of all possible outcomes of an experiment are called possibility set.
   c) Any sub set of outcomes of the experiment is called an event
2. a) Select one card randomly from the cards
   b) The sample space {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
   c) The card numbered with {1, 3, 5, 7, 9}

Answer for Activity 8.1

1. a) It is always happen(Certain)

   b) It is impossible
2. a) Certain

   b) Certain

   c) Impossible

Answer for Exercise 8.2

1. Choosing a yellow chocolate is impossible because there is no yellow chocolate in a glass jar
2. B
3. a) Impossible

   b) Certain

   c) Certain

Answer for Exercise 8.3

1. **a)** In rolling a dice, you will get 7 and When water boils it changes to milk

   b) In tossing a coin, you will get H or T

   c) Throwing a single die can produce six possible outcomes. All six outcomes are assumed equally likely. For any number of dice, the six faces are assumed equally likely to be produced and another example A cat is to chase a mouse.

   d) Your hair would grow 3 inches longer by tomorrow and A cat is to chase a dog
2. a)

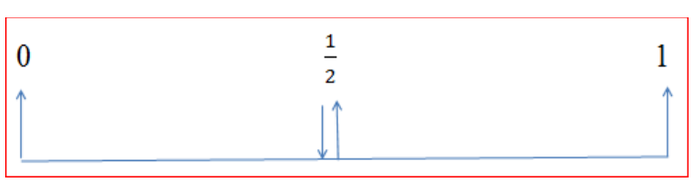


Figure 8.1

b) This will depend on the weather condition in July, it is reasonable to say it will rain at least one day on July in Addis Ababa is more likely.

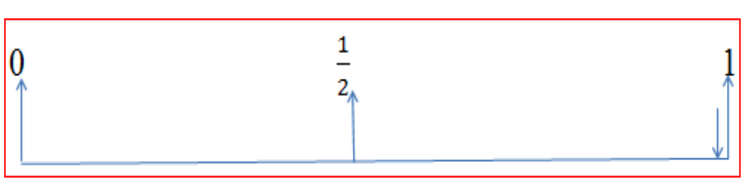


Figure 8.2

c) You will win a lottery is more unlikely the chance to win is closer to 0

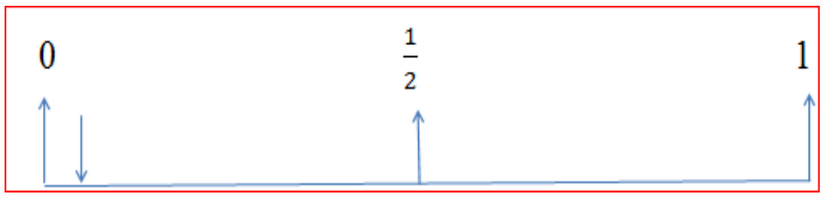


Figure 8.3

d) In a fair dice each face have equally likely to occur you expect from dice six equally likely

Figure 8.4

e) The probability of this is certain, as you can expect there to be birth day in this year

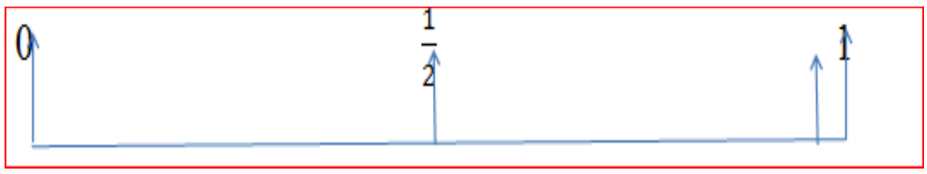


Figure 8.5

Assessment

You can assess students while they conduct experiments by way of asking them different oral questions such as: If you keep on doing an experiment of tossing a coin one hundred times how many times do you expect head to show up? You can also give Exercise 8.4 as a group work so that students can present their work. Or as this is the end of the sub- unit, you can give test/quiz to assess students learning and understanding.

Additionally ask your students the following question

- ❖ Ask students to identify certain, uncertain and impossible out comes
- ❖ Ask students to identify experiment, trial, event and sample space
- ❖ Ask students to solve real-life applications of examples of certain and impossible out comes.
- ❖ Provide descriptive feedback for learners to improve their learning

For slow learner students

Choose the correct answer from the given alternatives

1. Which one of the following best replaces the blank? 'A and B are playing tennis. The event of A or B winning the match is _____

   A, a likely event C, an unlikely event

   B, an unlikely event D, none

2. Teacher assigns home work after $\frac{7}{10}$ of the lessons

   A, unlikely B, likely C, Certain D, equally likely

3. There is a 0% chance that the earth is the closest planet to the sun

   A, unlikely B, likely C, Certain D, impossible

4. You miss a tree throw $\frac{3}{10}$ of the time

   A, likely B, unlikely C, Certain D, equally likely

5. A bag contains 5 red balls and 3 green balls. If a ball is chosen at random from the bag, then which of the following is an impossible event.

A, Choosing a red ball B, Choosing a yellow ball C, Choosing a green ball D, none

6. The measure of chance is known as

A) Geometry

B) statistics

C) probability

D) none of above

7. An event in the probability that will never be happened is called as -

A) Unsure event

B) Sure event

C) Possible event

D) Impossible event

For fast learner students

Choose the correct answer from the given alternatives

1. A teacher choosing one boy from a class of 19 girls is _____event

A, a possible B, a certain C, an impossible D, none

2. Your hair would grow 3 inches longer by tomorrow is _____events

A, a likely B, an unlikely C, equally likely D, a certain

3. You will eat tomorrow is_____ event

A, Certain B, an impossible C, possible D, none

4. Which of the following probability cannot exist?

A) 2/5

B) -1.5

C) 7

D) None of the above

5. What will be the probability of an impossible event?

A) 0

B) 1

C) Infinity

D) None of the above

6. Which of the following can be the probability of an event?

A) -1.3

B) 004

C) 3/8

D) 10/7

7. The probability of winning the first prize in a lottery of a girl is 8/100. If the 6000 tickets are sold, then how many tickets the girl purchased?

A) 480

B) 750

C) 280

D) None of the above

8. What will be the number of events if six coins are tossed simultaneously?

A) 32

B) 128

C) 100

D) 64

8.2. Probability of Simple events

Period Allotted: 7 periods

Competency: At the end of this sub-unit, students should:

- Find the probability of simple events.

Introduction

In this sub-topic, we will learn how to find the probability of a simple event.

The probability of an event is the likelihood of it occurring. When we discuss the likelihood of an event happening in everyday life, we may use some common words to describe this likelihood, for example, “certain”, “likely”, “very unlikely”, or “impossible”. In mathematics, we can assign a numerical value to a probability. Impossible events have a probability of zero, and events that are certain to happen have a probability of one. Events that are equally likely can be written with a probability of 0.5, or$\frac{1}{2}$. The sum of the probabilities of all possible outcomes must equal 1. For example, when flipping a coin, the probability of getting “heads” plus the probability of getting “tails” is one. This is because the probability of getting either one of heads or tails is certain, that is, a probability of one. In probability terms, a simple event refers to an event with a single outcome, for instance, getting “heads” with a single toss of a coin, or rolling a four on a die

Teaching notes

Events are equally likely to happen:

- When there is a 50% chance of rain, that means that there a chance that it might rain, but that there is also a chance that it might not rain. These chances are the same so the event is equally likely to happen.
- When one rolls a game die, he/she has exactly the same chance of landing on any of the six sides. Therefore, the probability of landing on any one specific side would be 1/6. This is also true for any spinner. Say a spinner is divided into 10 sections. Then there is an equally likely chance that the spinner can land on any of the sections. Thus, the probability for the spinner to land in any designated section is 1/10.
- The probability of selecting one of the three doors on the game show is also equally likely. There is no bias over the contestant’s decision so each door has

a probability of 1/3 being chosen. This is true if there are no arrows pointing towards one of the doors.

Answer for Exercise 8.4

1. a) P(E) = $\frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

   b) P (E) = $\frac{n(E)}{n(S)} = \frac{1}{6}$

   c) P (E) = $\frac{n(E)}{n(S)} = \frac{1}{6}$

   d) P (E) = $\frac{n(E)}{n(S)} = \frac{2}{6}$

   e) P (E) = $\frac{n(E)}{n(S)} = \frac{6}{6}$

2. a) n(letters I) = 2 and n(S) = 8 therefore, P(E) = $\frac{n(E)}{n(S)} = \frac{2}{8} = \frac{1}{4}$

   b) N (Vowels) = 5 and n(S) = 8 therefore, P(E) = $\frac{n(E)}{n(S)} = \frac{5}{8}$

   c) N (letters B) = 0 and n(S) = 8 therefore, P(E) = $\frac{n(E)}{n(S)} = \frac{0}{8} = 0$

3. The probability of zero means there is no chance to occur an event (impossible outcomes)
4. A

Assessments

You should asses each student's work continuously over the whole periods and ask the following question

- ❖ Ask group of students to derive the probability of simple Events
- ❖ Ask students to find the probability of simple events
- ❖ Ask students to solve real-life applications of probability of simple events.
- ❖ Provide descriptive feedback for learners to improve their learning

Finally give the following additional exercise problems for slow learner students and for fast learner students

For slow learner students

I choose the correct answer from the given alternatives

1. A bag contains 5 quarters, 2 dims, and 4 pennies, What is the probability of picking a quarters?

   A, $\frac{5}{11}$ B, $\frac{1}{3}$ C, $\frac{5}{6}$ D, 5

2. Find the experimental probability: Roll dice: 1, 3, 3, 4, 4 then P(1)=____

   A, $\frac{3}{5}$ B, $\frac{2}{5}$ C, $\frac{5}{6}$ D, $\frac{1}{5}$

3. If you cut a shuffled deck, what is the probability you will get an 8 of diamonds?

   A, $\frac{1}{4}$ B, $\frac{1}{52}$ C, $\frac{1}{13}$ D, $\frac{4}{52}$

4. Probability of a simple event is.

   A, what should happen B, what will happen

   C, what does happen D, what I want to happen

5. A card is drawn from a pack of 52 cards. What is the probability of getting a queen card?

   A) 1/26

   B) 1/52

   C) 3/13

   D) 1/13

6. A dice is rolled. The probability of getting a number x where $1 < x < 6$ is-

A) Greater than 0

B) Greater than 1

C) Between 1 and 0

D) Equal to 1

7. Which of the following statement is not true about probability?

   A) The probability of an impossible event is zero.

   B) Probability can be greater than one or less than zero.

   C) Probability cannot be greater than one.

   D) None of the above

Answer the following questions

8. A coin is tossed twice. Lists all the possible outcomes?
9. Two dice are rolled together. Lists all possible outcomes?
10. What is the probability of getting six on a roll of a pair of a dice?
11. What is the probability of rolling an even number on a die?

II Choose the correct answer

For fast learner students

12. If you rolled a 6 sided dice, what is the probability of a rolling a 3?

A, $\frac{1}{6}$ B, $\frac{2}{3}$ C, $\frac{3}{6}$ D, $\frac{5}{6}$

13. If you flipped 2 coins, what is the probability that both will land on tails?

A, $\frac{2}{4}$ B, $\frac{3}{4}$ C, $\frac{1}{4}$ D, $\frac{0}{4}$

14. A card is selected from a deck of playing cards, what is the probability of selecting a red card?

A, $\frac{1}{4}$ B, $\frac{3}{4}$ C, $\frac{0}{2}$ D, $\frac{1}{2}$

15. In a class, there are 12 boys and 16 girls. One of them is called out by an enrol number, what is the probability that the one called is a girl?

A, $\frac{1}{4}$ B, $\frac{2}{5}$ C, $\frac{5}{12}$ D, $\frac{4}{7}$

16. If two dice are thrown simultaneously, what is the probability of getting a multiple of 2 on one dice and multiple of 3 on the other dice?

A) 5/4

B) 5/12

C) 11/36

D) ½

17. A school has five houses named as A, B, C, D, and E. There are 23 students in a class in which 4 students are from house A, 8 students are from house B, 5 from C, 2 from D, and the rest from house E. Class teacher randomly selects a student to be the class monitor. What is the probability that the selected student is not from house A, B, and C?

A) 1/23

B) 2/23

C) 5/23

D) 6/23

Work out (show necessary steps)

18. Two dice are tossed, state the probability of each event
    a) The sum is 7 b) The sum is 13 c) The sum is not 7
19. When 4 coins are thrown what is the probability of
    a) 4 heads b) 2 heads and 2 tails c) at least one head

20. A coin is tossed and a die is rolled. What is the probability of obtaining
    a) a head and a 6 c) a tail and an odd number
    b) a head and an even number d) an even number
21. A machine used for a lottery has 10 balls. Each ball is marked with a different single digit from 0 to 9. One ball is randomly selected and the digit is the winning number. Use this data for question 1 to 6
    a) What is the probability that the number is greater than 6?
    b) What is the probability that the number is 3?
    c) What is the probability that the number is less than or equal to 3?
    d) What is the probability that the number is odd?
    e) What is the probability that the number is not 8?
    f) What is the probability that the number is 4 or 7?

8.3. Applications on Business, Climate, Road Transport, Accidents and Drug Effects

Period Allotted: 3 periods

Competency: At the end of this sub-unit, students should:

- Solve the probability of real-life problems.

Answer for Review Exercise unit 8

1. a) False b) True c) False d) False
2. a) A probability experiment is a situation where chance affects the result of an experiment. If the experiment can only have two outcomes, it is named Bernoulli trial. A coin flip is a probability experiment because chance affects whether a coin will land heads or tails when it is flipped.

   b) A sample space is a collection or a set of possible outcomes of a random experiment. The sample space is represented using the symbol, “S”. The sample space of an experiment is the set of all possible outcomes of that experiment

c) The difference between an outcome and an event is - an event is the total number of outcomes whereas an outcome is the result of an event. Consider an event of rolling a die · Sample space = {1, 2, 3, 4, 5, 6} · each no in the sample space is an outcome. There are six outcomes in this experiment

d) Getting an even number on the toss of a die and getting an odd number on the toss of a die are equally likely events, since the probabilities of each event are equal.

e), If an event cannot happen the probability of such an event is considered zero.

3. A
4. Sample space = {Monday, Tuesday, Wednesday, Thursday**,** Friday, Saturday, Sunday}, n(S) = 49
   a) n(E) = 3

      P (E) = $\frac{n(E)}{n(S)} = \frac{3}{49}$

   b) n(E) = 8

      P (E) = $\frac{n(E)}{n(S)} = \frac{8}{49}$

   c) n(E) = 0

      P (E) = $\frac{n(E)}{n(S)} = \frac{0}{49} = 0$
5. S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}

   n(S) = 30

   a) E = {6, 12, 18, 24, 30} , n(E) = 5

      P (A multiple of 6) = $\frac{n(E)}{n(S)} = \frac{5}{30} = \frac{1}{6}$

   b) E = {1, 4, 9, 16, 25} , n(E) = 5

      P (A perfect square) = $\frac{n(E)}{n(S)} = \frac{5}{30} = \frac{1}{6}$
6. S = {1, 2, 3, 4, 5, 6}, n(S) = 6
   a) E = {4}, n(E) = 1

      P (E) = $\frac{n(E)}{n(S)} = \frac{1}{6}$

   b) E = {2, 4, 6}, n(E) = 3

      P (E) = $\frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

   c) E = {5, 6}, n(E) = 2

      P (E) = $\frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

   d) E = {1, 2, 3, 4, 5, 6}, n(E) = 6

P (E) = $\frac{n(E)}{n(S)} = \frac{6}{6} = 1$

7. a) S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}, n(S) = 36

   b) E = {(1,6), (2,5), (3,4), (4,3), (5,5), (6,1)}, n(E) = 6

   P (E) = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

   c) E = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}, n(E) = 6

   P (E) = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

   d) E = {(1,2), (1,4), (1,6), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6},n(E)= 9

   P (E) = $\frac{n(E)}{n(S)} = \frac{9}{36} = \frac{1}{4}$

   e) E = {(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)}, n(E) = 6

   P (E) = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

8. S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18}, n(S) = 18

   a) E = {2, 4, 6, 8, 10, 12, 14, 16, 18}, n(E) = 9

   P (E) = $\frac{n(E)}{n(S)} = \frac{9}{18} = \frac{1}{2}$

   b) E = {3, 6, 9, 12, 15, 18}, n(E) = 6

   P (E) = $\frac{n(E)}{n(S)} = \frac{6}{18} = \frac{1}{3}$

   c) E = {2, 3, 5, 7, 11, 13, 17}, n(E) = 7

   P (E) = $\frac{n(E)}{n(S)} = \frac{7}{18}$

   d) E = {4, 8, 12, 16}, n(E) = 4

   P (E) = $\frac{n(E)}{n(S)} = \frac{4}{18} = \frac{2}{9}$

9. n(S) = 20

   a) n (E) = 13

   P (E) = $\frac{n(E)}{n(S)} = \frac{23}{20}$

   b) n (E) = 0

   P (E) = $\frac{n(E)}{n(S)} = \frac{0}{20} = 0$

   c)) n (E) = 20

   P (E) = $\frac{n(E)}{n(S)} = \frac{20}{20} = 1$

BIBLIOGRAPHY

- **Alemayehu Haile (phd) and Yismaw Alemu (phd)**
- **Asseged Atnafu: A comprehensive Guide to High School Mathematics.**
- **Yohannes Woldetensae (phd) – Basic Algebra with applications, Africa Beza college**
- **Larson H.Edwards: A Graphic A pproach (5th edition)**
- **Introduction to Coordinate Geometry: A guide for teachers; Module 29**
- **Geometry and Trigonometry: Peter H. Selby**
- **Ann X. Gantlet- Geometry.**
- **Ron Larson. Geometry**
- **Business Mathematics Higher secondary**
- **Sheldon H . Ross- Introductory Statistics (3rd edition)**
- **Bluman . Elementary Statistics (7th edition)**
- **Tussy. Basic Mathematics (4th edition)**
- **Introduction to Graphic Linear Equations**

Learning objectives for Grade 8

After completing grade 8, students should be able to:-

- Define and represent rational numbers as fractions
- Show the relationship among W, Z and Q.
- Apply operation of rational numbers
- Calculate the square and cube of rational numbers
- Determine square roots and cube roots of perfect squares and perfect cubes respectively.
- Extract approximate square roots of numbers by using the numerical table.
- Simplify algebraic expressions.
- Solve real life problems using variables.
- Draw equation of a line on a Cartesian coordinate plane.
- Solve linear inequalities by using rules of transformation.
- Give the conditions for triangles to be similar.
- Apply the tests for similarity to check whether two given triangles are similar or not.
- Give the relationships that exist between lines and circles.
- Apply basic facts about central and inscribed angles and angles formed by intersecting chords to solve related problems.
- Identify certain, uncertain and impossible outcomes.
- Describe event, sample space and probability of simple events.
- Calculate probabilities of simple events.
- Understand basic concepts about right angled triangles
- Apply important theorems on right angled triangles to solve related problems.

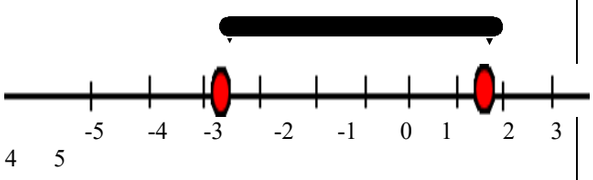
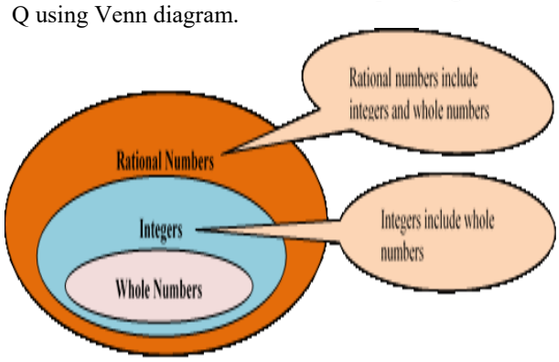
Unit 1: Rational numbers (30periods)

Learning Outcomes: At the end of this unit, learners will able to:

- Define and represent rational numbers as fractions
- Show the relationship among W, Z and Q.
- Order rational numbers.
- Solve problems involving addition, Subtraction, Multiplication and division of rational numbers
- Apply Rational Numbers to solve practical problems.
- Aware the four operations as they relate to Rational Numbers.

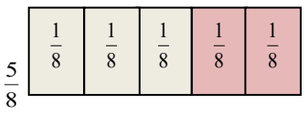
Competencies	Contents	Learning Strategies	Assessment
• Describe the concept of Rational Number practically • Express Rational numbers as fractions.	**1: Rational numbers** **1.1 The Concept of Rational Numbers (8 periods)**	• Encourage students to revise the order and representation of integers. • Divide students into small groups and let leaders present their ideas to the class for discussion. What is rational numbers? Discuss real life application of rational numbers? Eg. Solomon has 3 cats and 2 dogs. He wants to buy a toy for each of his pets. Solomon has 22 birr to spend on pet toys. How much can he spend on each pet? Write your answer as a fraction and as an amount in birr and cents. • Guide the students to solve the above problem using problem-solving methods as: **Solution: Understand the problem.**	• Observe learners while they are performing group work and present outcomes of their discussions • Record their performance • Provide constructive feedback based on their performance.

		The data: Total number of toys is 5 Total amount in birr is 22. The unknown: To find the amount spent for each pet. **Devise a plan** 1) Use a variable strategy i.e., let x be the amount spent for each pet. 2) Set up an equation. 5x = 22 **Carry out the plan** Solve the equation 5x = 22 $\Rightarrow$ 1/5 × 5x = 22 ×1/5 $\Rightarrow$ x = 22/5 or 4 birr and 40 cents. **Verifying the result** Check the result; 22/5 ×5 = 22 is correct.	

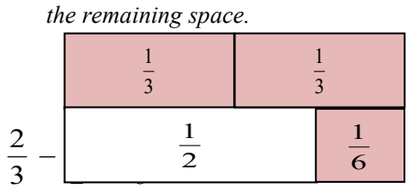
• Represent rational numbers as a set of fractions on a number line.	**1.1.1 Representation of Rational Numbers on a Number Line**	▪ Let students to represent some rational numbers on a number line. Eg. 1. Two rational numbers are opposites if they are the same distance from 0 but on different sides of 0. $2\frac{3}{4}$ and $-2\frac{3}{4}$ are opposites  2. **Analyze Relationships**: If you plot the point -8.85 on a number line, would you place it to the left or right of -8.8? Explain.	• Ask and let some students to represent a given rational n the number line. • Provide descriptive feedback for learners to impr learning.
• Describe the relationship among the sets W, Z and Q	**1.1.2 Relationship Among W, Z and Q**	• Let students to show the Relationship among W, Z and Q using Venn diagram.  • Give examples of set of numbers to show the relationship between rational numbers, integers, and whole numbers. ▪ Encourage the students to answers the questions like: 1). **Critique Reasoning**: Solomon says the number belongs only to the set of rational numbers. Explain his error. **2). Critical Thinking:** Is it possible for a number to be a rational number that is not an integer but is a whole number? Explain	• Ask and let some students to use venn diagram to relationship among W, Z and Q. • Provide descriptive feedback for learners to impr learning.
• Determine the	**1.1.3 Absolut**	• Ask students to tell the definition of the concept of absolute value of rational numbers " x"	• Ask and let some students to define and represen absolute value of a rational number on the number line.

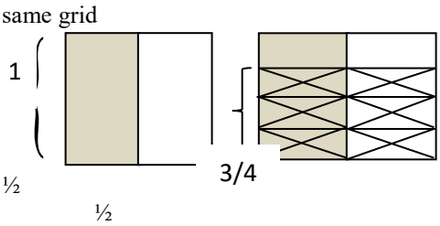
absolute value of a rational number	**e Value of Rational Numbers**	$\lvert x \rvert = \begin{cases} x & if\ x \geq 0 \\ x & if\ x < 0 \end{cases}$ • Engage students to interpret the concept of absolute value of a rational number geometrically. e.g $\lvert 3 \rvert = 3$ and $\lvert -3 \rvert = 3$ means that on a number line the distance of 3 from 0 is 3 units and the distance of −3 from 0 is also 3 units  • Encourage students to solve equation of absolute value like $\lvert x \rvert = 5$	• Provide constrictive feedback for learners to impr learning. • Ask and let some students to solve simple equation o value • Provide descriptive feedback for learners to impr learning. • Give homework exercise problems on evaluating absol of rational numbers

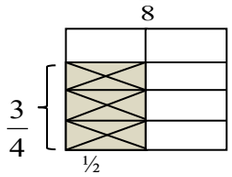
• Compare and order Rational numbers	**1.2 Comparing and Ordering Rational Numbers (8 periods)** **1.2.1 Comparing Rational Numbers** **1.2.2 Ordering Rational Numbers**	• Let students to compare rational numbers on a number line by changing the fractions to equivalent fractions with common denominator Eg. Use number line to compare decimals and fractions as: 1) Compare Decimals: Replace □ with <, >, or = to make a true statement. -1.2 □ 0.8 2) Compare Fractions: Replace □ with <, >, or = to make a true statement $\frac{-7}{8}$? $\frac{6}{8}$ Let students to order rational numbers on a number line by changing the fractions to equivalent fractions with common denominator o Eg. Use number line to compare decimals and fractions as: o Order 0.2, 0.8, and 0.4 from least to greatest. • Ask students individually or in groups a **real-life application** to order rational numbers like Samuel's science class is growing plants under different conditions. The average plant growth during a week was 5.5 cm. The table shows the difference from the average for some students' plants. Order the differences from lowest to highest. *Difference from Average Pl* *Student* | *Rahel* | *Kassa* | *Tesfay* *Difference (in.)* | $3\frac{1}{4}$ | *-2.2* | *1.7*	• Ask and let some students to compare and order rational numbers on the number line by changing the fractions to equivalent fractions with common denominator and by giving practical problems. • Provide constrictive feedback for learners to improve their learning. Give students various exercise problems to compare rational using the inequality signs "<" and ">" between given rational numbers and check their work. $-1\frac{7}{10}$

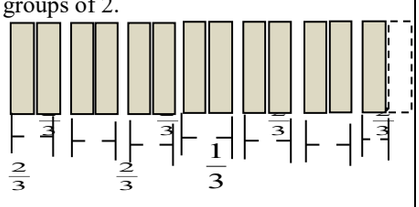
• Add rational numbers • Subtract rational numbers	**1.3 Operation and Properties of Rational Numbers (16 periods)** **1.3.1 Addition of Rational Numbers**	Let students revise addition of integers using a number line. • Divide students into groups and let them discuss by stating and showing the rule for addition of two rational numbers using examples • If the signs of the addends are different (i) take the sign of the addend with the greater absolute value. (ii) take the absolute values of both numbers and subtract the addend with smaller absolute value from the addend with greater absolute value. Eg. -6 + 2 = -4 • If both rational numbers are negative (i) decide (put) the sign first (ii) take the sum of the absolute values of the addend eg. -3 + (-5) = -8 • Let students show the validity of the commutative property and associative property. • Let students to generalize that, for any rational number X, Y, Z (i) X + Y = Y + Z (ii) (X + Y) + Z = X+ (Y + Z) • Encourage students to find addition of rational numbers using fraction bars. Eg. Find the sum of $\frac{3}{8}$ *and* $\frac{2}{8}$ using fraction bars.  $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$	• Ask students orally to answer addition of integers using number line. • Ask your students to find the addition of rational numbers a) using properties of addition b) by using fractional bars c) by using Scientific calculator • Ask students to solve real-life applications of addition. • Provide descriptive feedback for learners to improve their learning Give different exercise problems on addition of

	1.3.2 Subtraction of Rational Numbers	Let students to express subtraction of rational numbers as addition of the opposite of the rational number to be subtracted. **Example 1.** 5 − 4 = 5 + (-4) The rule studied for addition of rational	• Ask your students to find the subtraction of rational numbers a) using properties of subtraction b) by using fractional bars c) by using Scientific calculator

		numbers can be applied to find the sum 5 + (-4). Example 2. 5 - (-3) = 5 + 3 Encourage students to find subtraction of rational numbers using fraction bars. Eg. Find the differences $\frac{2}{3}$ - $\frac{1}{2}$ using fraction bars? *Place a $\frac{1}{2}$ bar beneath bars that show $\frac{2}{3}$, and find which fraction fills in the remaining space.*  	• Ask students to solve real-life applications of subtraction. • Provide descriptive feedback for learners to improve their learning • Give different exercise problems on subtraction of rational numbers and check their work. Give different exercise problems on the use of the commutative and associative properties and follow up the performance of students

• Multiply rational numbers • Divide rational numbers	**1.3.3 Multiplication of Rational Numbers**	• Divide students into ten groups and let them discuss on the rules for multiplication of rational numbers (i) Product of two rational numbers with different signs (a) decide the sign of the product, it is " - " (b) take the product of -(|-3 |× | 4| the absolute value the numbers **Example** -3 × 4 = - (|-3 | × |4 |) = - (3 × 4) = -12 (ii) Product of two negative rational numbers (a) decide the sign of the product, it is " + " (b) take the absolute values of the numbers and multiply them. **Example** (-3) × (-4) = ½ –3½ × ½ –4½ = 3 × 4 = 12 Engage students to come to the conclusion about the validity of the commutative, associative and distributive property of multiplication of rational numbers. • Encourage students to find multiplication of rational numbers using grids to model. Eg. Use a grid to model to find $\frac{3}{4} \times \frac{1}{2}$. Think of $\frac{3}{4}$ of $\frac{1}{2}$. a) Model $\frac{1}{2}$ by shading half of a grid b) Use a different colour to shade $\frac{3}{4}$ of the same grid  Divide the grid into 2 columns. Divide the grid into 4 rows. Shade 1 column to show $\frac{1}{2}$. Shade 3 rows to show $\frac{3}{4}$ c) Determine what fraction of the grid is shaded with both colours. There are 8 equal parts, and 3 of the parts are shaded with	• Ask your students to find the multiplication of Rational Numbers a) using properties of multiplication and division b) by using grids to models c) by using Scientific calculator • Ask students to solve real-life applications of multiplication and division • Provide descriptive feedback for learners to improve their learning • Give different exercise problems on multiplication and division of rational numbers and check their work. Give different exercise problems on the use of the commutative and associative properties and follow up the performance of students

		both colours. The fraction shaded with both colours is $\frac{3}{8}$.  $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ The section of the grid shaded with both colours shows 3 parts of $\frac{1}{2}$ when $\frac{1}{2}$ is divided into 4 equal parts. In other words, the grid shows $\frac{3}{4}$ of $\frac{1}{2}$, or $\frac{3}{4} \times \frac{1}{2}$	

	1.3.4 Division of Rational Numbers	Let students to discuss the rules for division of two rational numbers using examples and let the students come to the following conclusion. 1) To determine the sign of the quotient: - (a) If the signs of the dividend and the divisor are the same, the sign of the quotient is " + " you may take examples like: $\frac{-10}{-5} = \frac{10}{5} = 2$ • Encourage students to find division of rational numbers using grids to model. Eg. Use grids to model $4\frac{1}{3} \div \frac{2}{3}$. Divide 5 grids into thirds. Shade 4 grids and $\frac{1}{3}$ of a fifth grid to represent $4\frac{1}{3}$  *Think: How many groups of $\frac{2}{3}$ are in $4\frac{1}{3}$?* Divide the shaded grids into equal groups of 2.  	• Ask your students to find the division of Rational Numbers a) using properties of division b) by using grids to models c) by using Scientific calculator • Ask students to solve real-life applications of division • Provide descriptive feedback for learners to improve their learning • Give different exercise problems on division of rational numbers and check their work. • Give different exercise problems on the use of the commutative and associative properties and follow up the performance of students

Solve application of Rational Number	**1.4 Real-life Applications of Rational Numbers (5 periods)** **1.4.1 Application in sharing something among friends** **Application in calculating Interest and loans**	There are 6 groups of $\frac{2}{3}$, with $\frac{1}{3}$ left over. This piece is $\frac{1}{2}$ of a group of $\frac{2}{3}$. Thus, there are $6+\frac{1}{2}$ groups of $\frac{2}{3}$ in $4\frac{1}{3}$. $4\frac{1}{3}\div\frac{2}{3}=6+\frac{1}{2}$ • Guide students to solve **real-life applications** of rational numbers in sharing something among friends by **discussing in groups and present** in class. Guide students to solve **real-life applications** of rational numbers in calculating interest and loan	• Ask group of students to work the real-life application problems. • Observe students while they are working together • Ask your students to communicate their result • Provide descriptive feedback for learners to improve their learning

Unit 2: Squares, square roots, cubes and cube roots *(31 periods)*

Learning Outcomes: At the end of this unit, learners will be able to:

- Understand the notion square and square roots and cubes and cube roots
- Determine the square of numbers
- Determine the square roots of the perfect square numbers
- Extract the approximate square roots of numbers by using the numerical table and scientific calculator.
- Determine the cube of numbers
- Extract the cube roots of perfect cubes.
- Apply squares, square roots, cubes and cube roots in the real-life situation

• Calculate the square of a number	**2.1 Squares and Square Roots** ***(14 periods)*** 2.1.1 Square of a Rational Number 2.1.2 Use of table values and Scientific calculator to find squares of rational numbers	• Start the lesson by giving the **application of the unit**. • Engage the learners with warm-up activities by using rectangles to visualize the square of a number and then give examples of a square of a number like $9 \times 9 = 9^2 = 81$, and $\frac{1}{2} \times \frac{1}{2}$ or $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, which means squaring a number is multiplying the number by itself. • Guide group of students to **discuss and communicate** to approximate the square of numbers obtained from numerical table, i.e. $(2.25)^2 = 5.0625 \approx 5.06$ in the table. • Guide group of students to **discuss and communicate** with each other to apply **Scientific calculator** to find the squares of a number, • Motivate students by show interesting patterns of square of numbers and apply **critical thinking** to generalize, 1 [one odd number] $= 1 = 1^2$ $1 + 3$ [sum of first two odd numbers] $= 4 = 2^2$ $1 + 3 + 5$ [sum of first three odd numbers] $= 9 = 3^2$ $1 + 3 + 5 + 7$ [...] $= 16 = 4^2$ $1 + 3 + 5 + 7 + 9$ [...] $= 25 = 5^2$ $1 + 3 + 5 + 7 + 9 + 11$ [...] $= 36 = 6^2$ What is the *sum of first n odd natural numbers*?	• Ask students orally to answer simple squares of numbers. • Ask your students to find the square of a number and compare their result a) by computing b) by using table of square c) by using Scientific calculator • Ask students to solve real-life applications of square of numbers. • Provide descriptive feedback for learners to improve their learning

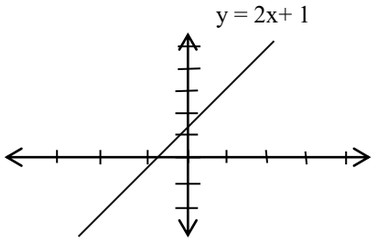
Competencies	Contents	Learning Strategies	Assessment
• Calculate the square of a number	**2.1 Squares and Square Roots** ***(14 periods)*** 2.1.1 Square of a Rational Number 2.1.2 Use of table values and Scientific calculator to find squares of rational numbers	• Start the lesson by giving the **application of the unit**. • Engage the learners with warm-up activities by using rectangles to visualize the square of a number and then give examples of a square of a number like $9 \times 9 = 9^2 = 81$, and $\frac{1}{2} \times \frac{1}{2}$ or $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, which means squaring a number is multiplying the number by itself. • Guide group of students to **discuss and communicate** to approximate the square of numbers obtained from numerical table, i.e. $(2.25)^2 = 5.0625 \approx 5.06$ in the table. • Guide group of students to **discuss and communicate** with each other to apply **Scientific calculator** to find the squares of a number, • Motivate students by show interesting patterns of square of numbers and apply **critical thinking** to generalize, 1 [one odd number] $= 1 = 1^2$ $1 + 3$ [sum of first two odd numbers] $= 4 = 2^2$ $1 + 3 + 5$ [sum of first three odd numbers] $= 9 = 3^2$ $1 + 3 + 5 + 7$ [...] $= 16 = 4^2$ $1 + 3 + 5 + 7 + 9$ [...] $= 25 = 5^2$ $1 + 3 + 5 + 7 + 9 + 11$ [...] $= 36 = 6^2$ What is the *sum of first n odd natural numbers*?	• Ask students orally to answer simple squares of numbers. • Ask your students to find the square of a number and compare their result a) by computing b) by using table of square c) by using Scientific calculator • Ask students to solve real-life applications of square of numbers. • Provide descriptive feedback for learners to improve their learning

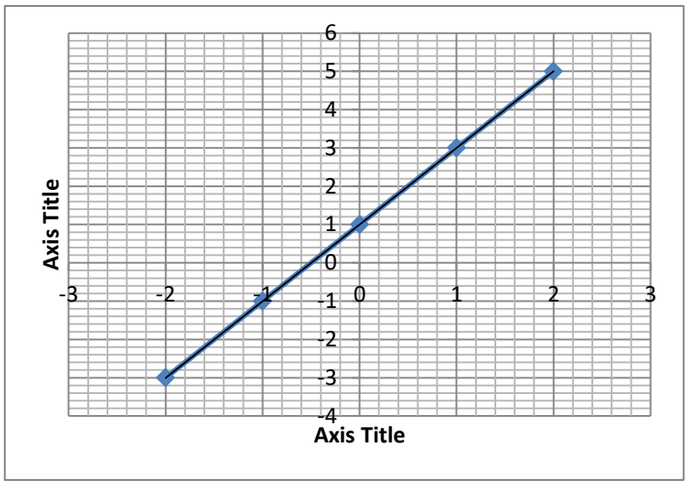
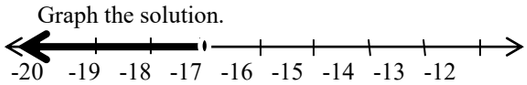
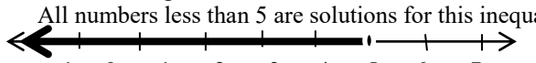
Competencies	Contents	Learning Strategies	Assessment
• Calculate the square root of perfect squares.	2.1.3 Square Roots of a Rational Number 2.1.4 Use of table values and Scientific calculator to find square roots of rational numbers	• Assist students to use **internet** and explain the relation between squaring and extracting square root from the square with examples like 2 squared = 2 × 2 = 4. Therefore, the square root of 4 is 2 • Assist also to identify the symbol $\sqrt{\ }$, called a *root sign,* represents a nonnegative square root. For example, $\sqrt{4} = 2$ is the positive square root of 4, and $-\sqrt{4} = -2$ is the negative square root of 4; but not $\sqrt{-4} = -2$ or 2. • Guide the student to use prime factorization to find the square root of perfect squares; for example, $\sqrt{100}$ 100 2 50 2 25 5 5 $100 = 2 \times 2 \times 5 \times 5 = (2 \times 5) \times (2 \times 5)$ $= 10 \times 10 = 10^2$ $\therefore \sqrt{100} = \sqrt{10^2} = 10$ • Guide students to solve **real-life applications** representing square of numbers discussing in groups and present in class **Remark**: Table of squares and square roots and using scientific calculator should be included in the text books.	• Give questions and ask students to find square root of non-negative numbers and compare their result a) by computing b) by using table of square c) by using Scientific calculator • Provide descriptive feedback for learners to improve their learning
• Calculate the cube of a number	**2.2 Cubes and Cube Roots** ***(14 periods)*** 2.2.1 Cube of a Rational Number	• Engage the learners with warm-up activities by using cubes to visualize the cube of a number and then give examples of a cube of a number like $3 \times 3 \times 3 = 3^3 = 27$, and $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ or $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$, which means cubing of a number is the product of three occurrences of itself. • Guide students to find some cubes of whole numbers using a table and **scientific calculator**. • Motivate students by show interesting patterns of cubes of numbers and apply **critical thinking** to	• Ask students orally to answer simple cubes of numbers. • Ask your students to find the cube of a number and compare their result a) by computing b) by using table of cube c) by using Scientific calculator • Ask students to solve real-life applications of cube of numbers. • Provide descriptive feedback for learners to improve their learning
	2.2.2 Cube Root of a Rational Number	generalize, Observe the following pattern of sums of odd numbers. $1 = 1 = 1^3$ $3 + 5 = 8 = 2^3$ $7 + 9 + 11 = 27 = 3^3$ $13 + 15 + 17 + 19 = 64 = 4^3$ $21 + 23 + 25 + 27 + 29 = 125 = 5^3$ How many consecutive odd numbers will be needed to obtain the sum as 10^3? • **Project as a critical thinking:** Give students as a project to think, discuss and write whether the following are perfect cubes. (i) 2700 (ii) 16000 (iii) 64000 (iv) 900 (v) 125000 (vi) 36000 (vii) 21600 (viii) 10,000 (ix) 27000000 (x) 1000. What pattern do you observe in these perfect cubes? • Assist students to use **internet** and explain the relation between cubing and extracting cube root from the cube of a number with examples like 2 cubed = 2×2×2 = 8. Therefore, the cube root of 8 is 2. • Assist also to identify the symbol $\sqrt[3]{\ }$, called a *cube root*. For example, $\sqrt[3]{8} = 2$; similarly, $\sqrt[3]{-8} = \sqrt[3]{-2 \times -2 \times -2} = -2$.	
• Solve real-life problems	**2.3 Applications on squares, square roots, cubes and cube roots** ***(3 periods)***	• Guide students to solve **real-life applications** on Application in carpentry, Application in architecting, and Application in engineering, representing cubes of numbers **discussing in groups and present** in class.	• Ask group of students to work the real-life application problems. • Observe students while they are working together • Ask your students to communicate their result • Provide descriptive feedback for learners to improve their learning

Unit 3: Linear equation and inequalities *(26 periods)*

Learning Outcomes: At the end of this unit, learners will able to:

- Graph linear equations of type y = m*x* +n
- Solve linear inequalities
- Solve applications of linear inequalities

<table>
<tr><th>Competencies</th><th>Contents</th><th>Learning Strategies</th><th>Assessment</th></tr>
<tr><td>• Describe the Cartesian coordinate system</td><td>3.1 Revision of Cartesian Coordinate System (2 periods)</td><td>• Assist students to revise the x - axis and y-axis divide the coordinate plane into four quadrants and to read and plot points in the coordinate’s plane.
• Guide students to describe the sign of coordinates (x, y) in each quadrant.</td><td>• Ask students to identif the quadrants of the Cartesian plane.
• Ask students to locate the coordinates in each quadrant.</td></tr>
<tr><td>• Draw linear equations like y = mx + n in a Cartesian coordinate plane</td><td>3.2 Graph of linear equations (10 periods)</td><td>• Revise to the students to draw a vertical line and come to the conclusion that x = b where b ∈ Q is an equation for a vertical line.
• Assist students to draw a line whose equation is of the form y = mx + n, where m, n ∈ Q by the following steps:
1. Make table of values for easy x coordinates
2. Use the equation y = mx + n to calculate the y value
3. Plot the points
4. Draw the lines through these points
Example: Draw the line y = 2x + 1
<table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>-3</td><td>-1</td><td>1</td><td>3</td><td>5</td></tr></table>y = 2x+ 1
 
• Guide students to draw the graph of y = 2x + 1 by using software such as computer Excel or Geogrba or Mathlab or Mathematica, etc and compare your result with manual drawing. Below is the graph of y = 2x using computer Excel</td><td>• .Ask your students to draw linear equations like y = mx + n; in a Cartesian coordinate plane and compare their result
a) by using table value manually
b) by using computer applications
• Provide descriptiv feedback for learne to improve the learning</td></tr>
</table>

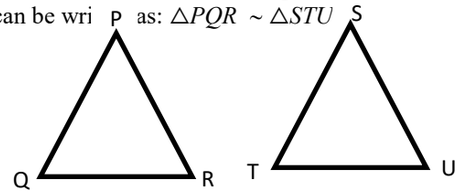
Competencies	Contents	Learning Strategies	Assessment
		 • Assist students to discuss in groups and communicate to graph y = 2x + 3, y = 3x + 5. Observe that the natures of the graphs are similar.	
• Solve linear inequalities	**3.3 Solving Linear Inequalities** ***(10 periods)***	• Guide students to discuss in groups and communicate the difference between x > 5 and x ≥ 5; x > 5 and x < 5; and x ≥ 5 and x ≤ 5, etc. • Assist students to solve and graph the solution of One-Step Inequalities, such as **Example:** Solve each inequality. Graph and check the solution. $x + 5 < -12$ Solve the inequality. $x < -17$ Graph the solution.  -20 -19 -18 -17 -16 -15 -14 -13 -12 **Check the solution.** Substitute a solution from the shaded part of your number line into the original inequality. -18 + 5 ? < -12 -13 < -12**EXA** • Assist students to solve and graph the solution of two steps inequalities, such as.	• Ask your students to solve linear inequalities. • Ask students to solve real-life applications o linear inequalities. • Provide descriptiv feedback for learners t improve their learning
		Example: Solve and graph the solution of $-3x + 7 > -8$. $-3x + 7 > -8$ $-3x > -7 - 8$ $-3x > -15$ $x < 5$, because dividing or multiplying by negative makes the sign changed. All numbers less than 5 are solutions for this inequality.  -1 0 1 2 3 4 5 6 7 • Assist students to discuss in groups and communicate the following **Critical Thinking.** Selam scored 95, 86, and 89 on three science tests. She wants her average score for 6 tests to be at least 90. What inequality can you write to find the average scores that she can get on her next three tests to meet this goal? Use x to represent the lowest average score. • Assist students to discuss in groups and communicate the following **Critical Thinking.** Is there any value of x with the property that $x < x - 1$? Explain your reasoning. • Assist students to solve real-life problems to solve linear inequalities.	
• Apply linear equations and inequalities in the real-life situation • Solve linear equations and inequalities real-life problems	**3.4 Applications in Linear Equations and Inequalities** ***(4 periods)***	• Assist students to work in groups and communicate the solutions of the word problems in agriculture, engineering, business, education, day to day activities, etc. • Guide students to discuss and in groups and communicate the solution of the problems like road transport, business, etc. • Assist students to discuss in groups on causes and effects of climate change, and protection of the environment • Guide students to solve a real-life application problems like the Rise of Atmospheric Carbon Dioxide; Rising of Temperature; Rise of Sea Level, etc.	• Ask group of students to solve real-life application problems i terms of linear equations and inequalities. • Observe students whi they are workin together • Ask your students to communicate their result

Competencies	Contents	Learning Strategies	Assessment
			• Provide descriptive feedback for learners t improve their learning

Unit 4: Similarity of Figures *(18 periods)*

Learning Outcomes: At the end of this unit, learners will able to:

- Know the concept of similar figures and related terminologies
- Understand the condition for triangles being similar.
- Apply tests to check whether two given triangles are similar or not.
- Apply real-life situations in solving geometric problems

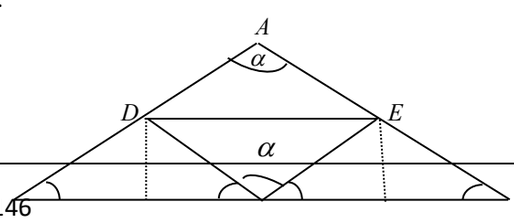
Competencies	Contents	Learning Strategies	Assessment
• Identify figures that are similar to each other • Apply the definition of similarity of two triangles to solve related problems.	**4.1 Similar Plane Figures *(10 periods)*** **4.1.1 Definition and Illustration of Similar Figures** **4.1.2 Similar Triangles**	• Aware the students about the concept of similar figures by using models of figures or objects like: photographs, polygons having the same shape but not necessarily the same size. • Assist students in groups to draw different pairs of similar figures and to give examples of similar figures from their everyday life. • Help the students to discuss in groups and communicate the following as **Critical Thinking:** In two similar triangles: o the measures of their corresponding angles are equal, and o the lengths of their corresponding sides are proportional. • **Critical Thinking:** Assist students to realize that it is not necessary to compare the two conditions; the two conditions are equivalent. • Students should aware that the symbol ~ means "is similar to." Two similar triangles $\triangle PQR$ and $\triangle STU$ can be wri as: $\triangle PQR \sim \triangle STU$  If $\triangle PQR \sim \triangle STU$ then $\angle P \cong \angle S$, $\angle Q \cong \angle T$, and $\angle R \cong \angle U$, and $\frac{\overline{PQ}}{\overline{ST}} = \frac{\overline{QR}}{\overline{TU}} = \frac{\overline{PR}}{\overline{SU}}$. • Assist students to solve problems like, Given $\triangle PQR \sim \triangle STU$, and $\overline{PQ}$ = 6 cm, $\overline{TU}$ = 3 cm, $\overline{PR}$ = 12 cm, and $\overline{SU}$ = 4 cm then find all sides of $\triangle PQR$ and $\triangle STU$? $\frac{\overline{PQ}}{\overline{ST}} = \frac{\overline{QR}}{\overline{TU}} = \frac{\overline{PR}}{\overline{SU}} \Rightarrow \frac{6}{\overline{ST}} = \frac{\overline{QR}}{3} = \frac{12}{4} = 3$ $\Rightarrow \overline{ST}$ = 2 cm and $\overline{QR}$ = 9 cm	• Ask students to identify the similarity of two given figures • Ask students to explain the concept of similar figures • Ask students to solve problems using two similar triangles • Provide descriptive feedback for learners to improve their learning

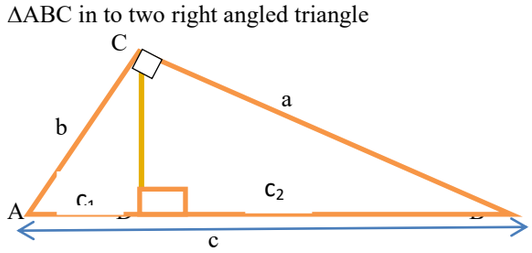
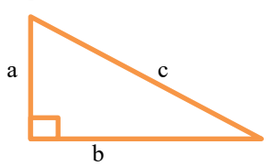
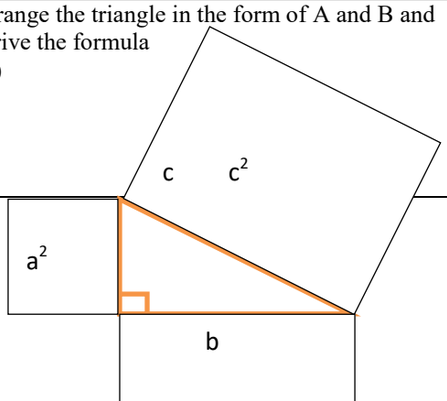
Competencies	Contents	Learning Strategies	Assessment
• Determine the similarity of two triangles	**4.1.3 Tests for similarity of triangles (AA, SSS, and SAS)**	• Guide the students to answer the following: In the triangles ΔPQR *and* $\triangle STU$, if $\angle P \cong \angle S$, and $\angle Q \cong \angle T$. What must be true about $\angle R$ and $\angle U$? Why? • **Critical Thinking:** Help students in groups to draw a triangle ΔPQR. Use a compass and straightedge to construct a second triangle, $\triangle STU$, with $\angle P \cong \angle S$, and $\angle Q \cong \angle T$. Are your triangles similar? Explain. This answer the AA similarity. • **Critical Thinking:** Help students in groups to construct another pair of triangles so that the side lengths of one triangle are a multiple of the side lengths of the other. Compare the corresponding angles of your triangles. Your work in this investigation should support SSS similarity conjecture. **Note:** If AA is a similarity shortcut, then so are ASA, SAA, and AAA, because each of those shortcuts contains two angles. That leaves SAS and SSA as possible shortcuts. In the next investigation, you will look at SAS. • **Critical Thinking:** Help students in groups to construct two different triangles that have two pairs of sides proportional and the pair of included angles equal in measure. Can you do it? Your findings should support SAS similarity conjecture.	• Ask students to describe each of the similarity tests of triangles AA, SSS, and SAS • Ask students to determine the similarity of two triangles by using the SSS, SAS and AA similarity tests for triangles • Provide descriptive feedback for learners to improve their learning
• Explain the relation between the perimeters of two similar triangles • Explain the relation between the areas of two similar triangles	**4.1.4 Perimeter and Area of Similar Triangles** ***(8 periods)***	• Help the students to revise the formula for perimeters and areas of triangles; and find perimeters and areas of two similar triangles and their ratios of the perimeters and areas. • **Critical Thinking:** Help students in groups to conclude "the ratio of the perimeters of two similar triangles is equal to the ratio of their corresponding sides" and "The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides."	• Ask students to relate the perimeters of two similar triangles • Ask students to relate the areas of two similar triangles • Provide descriptive feedback for learners to improve their learning

Unit 5: Theorems on Triangles *(19 periods)*

Learning Outcomes: At the end of this unit, learners will able to:

- Understand basic concepts about right angled triangles
- Apply some important theorems on right angled triangles.
- Apply real-life situations in solving geometric problems

Competencies	Contents	Learning Strategies	Assessment
• Describe the angle sum theorem of a triangle • Apply the angle sum theorem of a triangle in solving related problems.	**5 Theorems on Triangles** **5.1 The three angles of a triangle add up to 180°** ***(3 periods)***	• Guide the students in group work to draw any triangle, cut it out carefully, tear the vertices off and fit them together. Based on the result lead them to communicate their result that the sum of the measures of interior angles of a triangle is 180^0. • Another activity for group of students was, fold AB to get the mid-point at D and fold AC to get the mid- point at E. From D and E draw perpendicular line to line segment BC. Fold in wards ΔADE at line segment DE, ΔDBF at line segment DF and ΔCEG at line segment EG. The three angles α, β and θ added together are seen to form a straight line on the base, that is, they are equal to 180°. 	• Ask students to describe the angle sum theorem of a triangle • Ask students to prove the sum of the measures of interior angles of a triangle is 180^0 using paper folding • Ask students to solve problems using the angle sum theorem of a triangle • Provide descriptiv feedback for learner to improve the learning

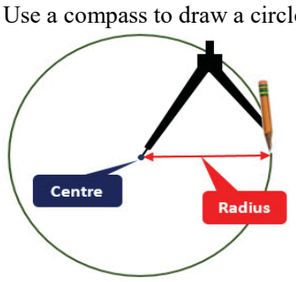
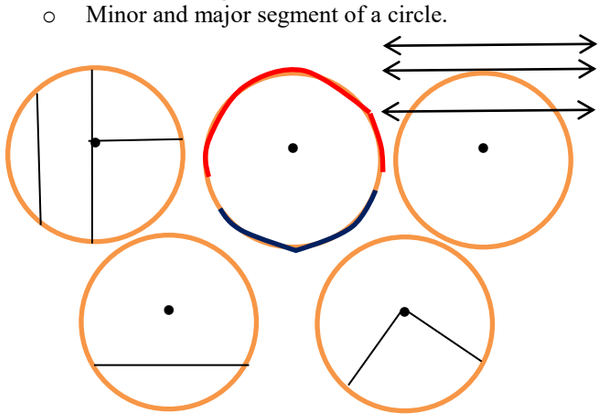
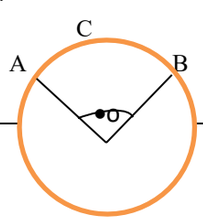
Competencies	Contents	Learning Strategies	Assessment
		β β θ θ *B* *F* *G* *C*	
• Describe the relation between the exterior angle and the two remote interior angles of a triangle. • Prove the exterior angle of a triangle equals the sum of the two remote interior angles	**5.2 The exterior angle of a triangle equals the sum of the two remote interior angles.** ***(4 periods)***	• Assist students to prove that the sum of degree measure of interior angles of a triangle is 180^0 based on the knowledge of alternate interior angles and the measure of straight angle which is 180^0. • Finally, guide the students to find the unknown angle of a triangle by applying the angle sum theorem of a triangle. • Encourage the students to prove the exterior angles theorem of triangle by using the knowledge of angle sum theorem of a triangle and the measure of straight angle. • Finally, guide the students to determine unknown interior or exterior angle of a triangle by applying the exterior angle theorem of triangle.	• Ask students to describe the exterior angle and the two remote interior angles of a triangle • Ask students to prove the exterior angle of a triangle equals the sum of the two remote interior angles • Ask students to solve problems using the exterior angle theorem of triangle • Provide descriptiv feedback for learner to improve the learning
• Describe the right angle triangle, the altitude and hypotenuse • Apply Euclid's theorem and its converse for solving related problems	**5.3 Theorems on the right angled triangle** ***(12 periods)*** **5.3.1 Euclid' Theorem and its Converse**	• Start the lesson by considering a right angled triangle ΔABC and CD is its altitude, and the altitude divides ΔABC in to two right angled triangle  • Guide students to find similar triangles from ΔABC with the other two triangles formed and have the following similarities i) ΔCBD ~ ΔABC... (by AA similarity test) $\frac{CB}{AB} = \frac{BD}{BC} \Rightarrow \frac{a}{c} = \frac{c_2}{a} \Rightarrow a^2 = cc_2$ ii) D ACD ~ DABC - (by AA similarity test) $\frac{AC}{AB} = \frac{AD}{AC} \Rightarrow \frac{b}{c} = \frac{c_1}{b} \Rightarrow b^2 = cc_1$ • Following this you can state the Euclid's Theorem and its converse • Encourage students to apply the theorem in exercises like: **Example.** DABC is a right angled triangle with hypotenuse AB, and altitude CD to AB. If AD = 4 cm, DB = 5cm, find the lengths of AC and BC • **Critical Thinking:** Similarly you can give examples to illustrate the converse of the theorem.	• Ask students to describe the right angle triangle, the altitude and hypotenuse • Ask students to solve problems using Euclid's theorem and its converse • Provide descriptive feedback for learners to improve their learning
• Derive the Pythagoras theorem by using Euclid's theorem and paper folding • Apply Pythagoras' Theorem and its converse for solving related problem.	**5.3.2 The Pythagoras' theorem and its converse**	• Help students to revise Euclid's Theorem • Assist students to use the Euclidean relation to derive the Pythagorean relation and then state the theorem. • Guide students to derive the Pythagoras formula by different approaches: Let consider the right angle triangle in paper folding seen below  	• Ask students to derive the Pythagoras theorem by using Euclid's theorem and paper folding • Ask students to solve problems using Pythagoras' Theorem and its converse • Provide descriptive feedback for learners to improve their learning
		Arrange the triangle in the form of A and B and derive the formula A)  	

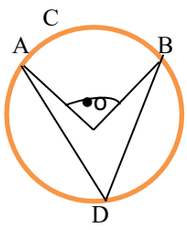
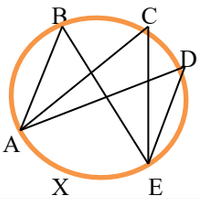
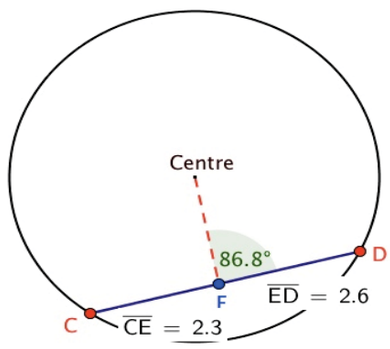
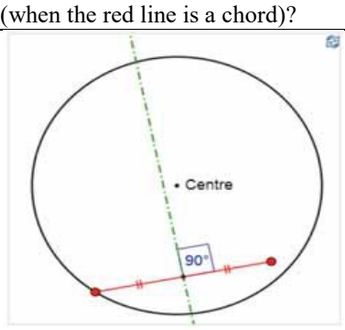
Competencies	Contents	Learning Strategies	Assessment
		a b B) a b b c c a • Encourage students to apply the "Pythagoras" Theorem to solve a real world problem.	

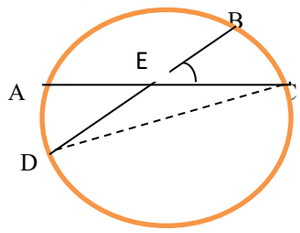
Unit 6: Lines and Angles in a Circle *(21 periods)*

Learning Outcomes: At the end of this unit, learners will able to:

- Have a better understanding of circles
- Realize the relationship between lines and circles
- Apply basic facts about central and inscribed angles and angles formed by intersecting chords to compute their measures.
- Apply real-life situations in solving geometric problems

Competencies	Contents	Learning Strategies	Assessment
• Identify the different types of arcs, sectors, segments • Describe the concepts "tangent" and "secant" of a circle.	**6.1 Circles *(18 periods)*** **6.1.1 Lines and Circles**	• Use a compass to draw a circle.  • Guide students to differentiate by giving examples between ○ Radius, diameter, chard, secant and tangent of a circle. ○ Minor and major arc of a circle. ○ Minor and major sector. ○ Minor and major segment of a circle. 	• Ask students to describe the concepts in a circle. • Ask students to identify major arc and minor arc. • Ask students to identify major and minor sector of a circle. • Ask students to identify major and minor segment of a circle. • Ask students to describe the concepts "tangent" and "secant" of a circle. • Provide descriptive feedback for learners to improve their learning
• Describe the central angles and inscribed angles • Find the measure of central angle or inscribed angle or	**6.1.2 Central angle and inscribed**	• Guide students to relate the central angle with the arc subtending it.  	• Ask students to describe the central angles and inscribed angles. • Ask students to identify central angles and

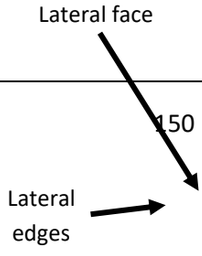
Competencies	Contents	Learning Strategies	Assessment
the intercepted arc based on the given information.	**angle**	o $m(\angle AOB) = m(\overset{\frown}{ACB})$ • **Critical Thinking:** After introducing the meaning of inscribed angle, encourage the students to measure the central angle and the inscribed angle subtended by the same arc and conclude that 1. The measure of the inscribed angle is half of the measure of central angle 2. The measure of the inscribed angle is half of the measure of the arc subtends it.  $m(\angle ADB) = \frac{1}{2}\, m(\angle AOB)$ $m(\angle ADB) = \frac{1}{2}\, m(\overset{\frown}{AOB})$ • **Critical Thinking:** Let students relate inscribed angles subtended by the same arc. i.e. $m(\angle ABE) = m(\angle ACE) = m(\angle ADE)$ $= \frac{1}{2} m(AXE)$  	inscribed angles. • Ask students to find the measure of central angle or inscribed angle or the intercepted arc based on the given information.. • Provide descriptive feedback for learners to improve their learning
		• **Project as a critical thinking:** This investigation is about a line drawn from the centre to a chord. *Drag the point E.* What do you notice about the length of CE and DE when the angle is 90°? Change the chord by dragging C and D and repeat the process.  Complete the statement: If the angle is 90° then CE=… Drag the point *E* in the previous sketch. What do you notice about the angle if *CE* = *DE*? • **Project as a critical thinking:** What do you notice about the perpendicular bisector when both of the red points are on the circle (when the red line is a chord)?	•
• Describe the angle formed by two intersecting chords • Solve problems related to angle formed by two intersecting chords inside a circle.	**6.1.3 Angles formed by two intersecting chords**	 We refer to a new discovery such as: *If the perpendicular bisector of a chord is drawn, then it passes through the centre of the circle.*	• Ask students to describe the angle formed by two intersecting chords. • Ask students to prove that an angle formed by two chords intersecting inside a circle is equal to half the sum of the intercepted arc by measurement. • Ask students to solve problems related to angle formed by two intersecting chords

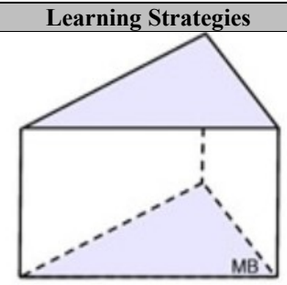
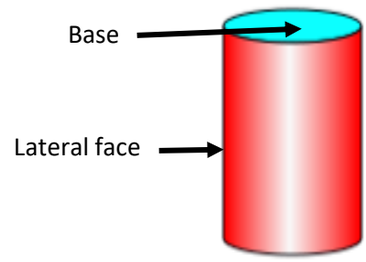
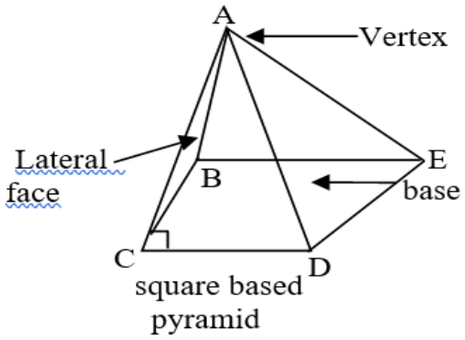
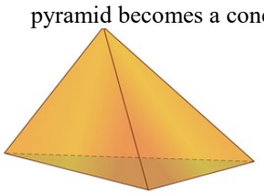
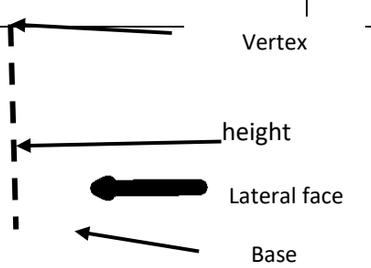
Competencies	Contents	Learning Strategies	Assessment
			inside a circle. • Provide descriptive feedback for learners to improve their learning
		• Assist students to conclude the following as discovery: ○ *The centre of a circle is on the perpendicular bisector of any chord, therefore their intersection point is the centre.* ○ *The perpendicular bisectors of the sides of a triangle meet at the centre of the circumscribed circle.* ○ *The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle.* ○ *The angle subtended on the circle by the diameter is always 90°.* • **Critical Thinking:** Given the circle below and lead the students  1. To connect D & C using ruler 2. To measure ∠ACD and ∠BDC using protractor 3. Based on step 2 ask them to write the measures of $\widehat{AD}$ & $\widehat{BC}$ 4. To measure ∠BEC 5. To relate m (∠BEC) with the sum of m($\widehat{AD}$) & m($\widehat{BC}$) That is m (∠BEC) = ½[m ($\widehat{AD}$) + m ($\widehat{BC}$)	
• Solve the application problems	**6.2 Applications of Circle *(3 periods)***	• Assist students to work in groups and communicate the results of the real-life applications in nature surrounding humans, technology, architecture, and mappings.	• Ask group of students to solve the real-life application problems in terms of geometry. • Observe students while they are working together • Ask your students to communicate their resul • Provide descriptive feedback for learners to improve their learning

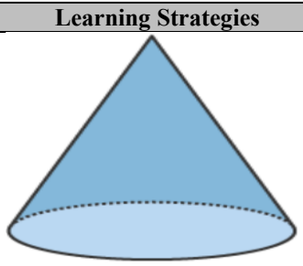
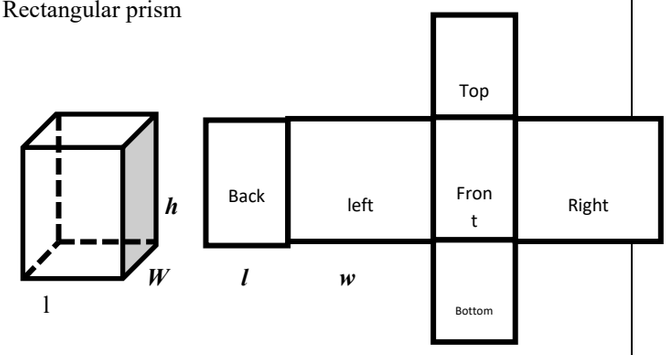
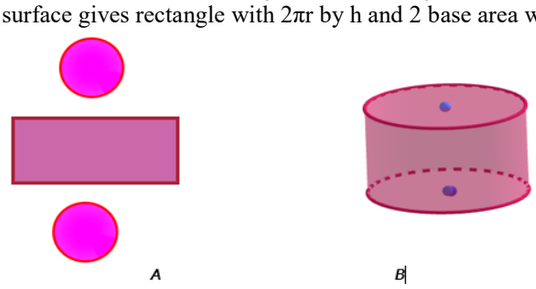
Unit 7: Solid Figures and Measurement (25 periods)

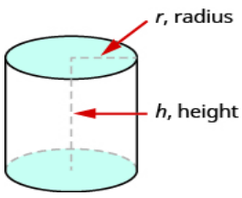
Learning Outcomes: At the end of this unit, learners will able to:

- Identify parts of solid figures
- Find the surface area of solid figures
- Find the volume of solid figures
- Solve applications of solid figures and measurements

Competencies	Contents	Learning Strategies	Assessment
• Identify parts of a prisms, cylinders, pyramid and cones • Name different types of prisms, cylinders, pyramid and cones based on their bases.	**7.1 Solid Figures *(8 periods)*** **7.1.1 Prisms and Cylinders**	• Divide students into small groups and let them to mention different objects having the shape of prisms by showing models of prisms and net of prisms. • Encourage students to identify bases, edge, vertices and faces of prisms from the model. • Encourage the students to define a prism as follows, A prism is a three-dimensional solid figure with two parallel faces, called bases, that are congruent triangle, square and rectangle, lateral edges and lateral flat face which are rectangles in a right prism. The prism takes its name from the name of the base  	Ask questions and let son students to identify parts prisn and cylinders by net of prisn and cylinders. a) using Geogerba Software b) using manually. • Provide constrictive feedba for learners to improve the learning. ➢ Give project on the pa of prisms and cylinders and t teacher checks the performan

Competencies	Contents	Learning Strategies	Assessment
		 Right Triangular Prism • Divide students into small groups and let them to introduce different types of prisms like rectangular prisms, square prisms and conclude that when the base is a circle the prism becomes cylinder.  We can see here that the net of a rectangular prism is made from six rectangles.	of students.
	7.1.2 Pyramids and Cones	• Divide students into small groups and let them to mention different objects having the shape of pyramid by showing models of pyramid. • Encourage students to identify vertex, edge and faces of pyramids from the model. • Encourage the students to define a pyramid as follows, A pyramid is a solid defined by a base and a point, called an apex, not on the base. The pyramid takes its name from the name of the base.  • Divide students into small groups and let them to introduce different types of pyramids like triangular pyramid, rectangular pyramid and conclude that when the base is a circle the pyramid becomes a cone.  	• Ask questions and let son students to identify par Pyramids and Cones by net Pyramids and Cones a) using Geogerba Software b) using manually. • Provide constrictive feedba for learners to improve the learning. • Give project on the parts Pyramids and Cones and t teacher checks the performan of students

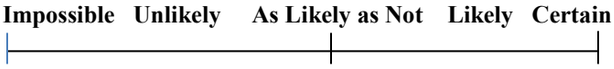
Competencies	Contents	Learning Strategies	Assessment
		 • Divide students into small groups and let them to derive the surface area formula of right prism by using net of prisms. Eg. Rectangular prism  	
• Find the surface area of prisms, cylinders, pyramid and cone	**7.2 Surface Area and Volume of Solid Figures (14 periods)** **7.2.1 Surface Area of Prisms, Pyramid and Cylinder**	• Guide the students to see the net of a rectangular prism is made from six rectangles, and the front and the back are the same, the right side and the left side are the same, and the top and the bottom are the same Area of front = Area of back = lh Area of left = Area of Right = wh Area of top = Area of bottom = lw • Lateral surface Area (A_l) = Sum of areas of lateral faces = Area of front + Area of back + Area of left + Area of right = lh + lh + wh + w h =2lh + 2wh = 2h (l + w) OR = h × 2 (l + w) = ph (p = perimeter of the base) Total surface Area (A_T) = Al + $2A_B$ = lateral surface area + Area of two bases (upper & lower) = 2lh + 2wh + 2lw = 2(lh + wh + lw) ∴ Total surface area = A_l + $2A_B$ • Let students use the same formula for finding Al of triangular prism. • Encourage students to apply the formula for computing the surface area of different prism. • Divide students into small groups and let them to derive the surface area formula of cylinder. • First let students to cut a cylinder vertically. Then its lateral surface gives rectangle with 2πr by h and 2 base area with πr^2.  Al = 2πr × h = 2πrh A_T = A_l + $2A_B$ A_T = (Circumference of bases) × height + 2(Area of bases) A_T = $2\pi rh + 2\pi r^2$ A_T = 2πr(h + r) • Encourage students to apply the formula for computing the surface area of cylinder.	• Ask questions and let son students how to derive t surface area formula of cylind by net of cylinder. a) using Geogerba Software b) using manually. • Provide constrictive feedba for learners to improve the learning. ➢ Give project on t surface area of cylinder and t teacher checks the performan of students.
• Find the volume of prisms and cylinders	**7.2.2 Volume**	• Encourage students to elaborate a common volume formula for the right prisms	• Ask questions and let son students how to derive t volume of prisms and cylind

Competencies	Contents	Learning Strategies	Assessment
	of Prisms and Cylinders	 h base V = Base Area x height $V = A_B h$ • Guide students to reach the conclusion that a cylinder is a circular prism. Thus, its volume can be found using same formula as all prisms  $V = A_B h$ But its base is a circle, $A_B = \pi r^2$ $V = \pi r^2 h$ • Guide students to apply the formula for computing the volumes of prisms and cylinder • Encourage the students to use the knowledge of geometry and measurements in fencing off an area to plot a crop, Planning the construction of a house, building a swimming pool, fill up your vehicle, the **volume** of gasoline your gas tank holds determines your purchase	by net of prisms and cylinder a) using Geogerba Software b) using manually. • Provide constrictive feedba for learners to improve th learning. ➢ Give activity on the volume prisms and cylinder and t teacher checks the performan of students.
• Solve applications of solid figures and measurements	**7.3 Applications on Solid Figures and Measurements *(3 periods)***	Give applicable problems in the area of Fencing off an **area** to plot a crop, Planning the construction of a house, building a swimming pool and Fuelling Up, when you fill up your vehicle, the **volume** of gasoline your gas tank holds determines your purchase.	• Ask group of students to wo the application problems. • Observe students while they a working together • Ask your students to communica their result •Provide descriptive feedback f learners to improve th learning

Unit 8: Introduction to Probability *(18 periods)*

Learning Outcomes: At the end of this unit, learners will able to:

- Understand the concept of probability
- Find event, sample space and probability of simple events.
- Apply problems of real-life situations in solving the probabilities

Competencies	Contents	Learning Strategies	Assessment
• Describe the concepts of probability	**8.1 The Concept of Probability *(6 periods)***	• Start the lesson by providing the application of the unit. • Assist students to use **internet** and describe the basic concepts of probability such as experiment, trial, outcomes, and event. An **experiment** is an activity involving chance in which results are observed. Each observation of an experiment is a **trial**, and each result is an **outcome,** the set of all possible outcomes of the experiment is the **sample space** of the experiment. A set of one or more outcomes is an **event**. If the event is not likely to occur, the probability of the event is close to 0. If an event is likely to occur, the event's probability is closer to 1. **Impossible Unlikely As Likely as Not Likely Certain**  0 $\frac{1}{2}$ 1 • Assist students to give real-life examples of certain and impossible out comes. • **Project as a critical thinking:** Assist group of the students to experiment the tossing of coins repeatedly (say up to 20, 30, 50, . . .) and register the outcomes. What do a group of students suggest the number of observing or getting head or tail?	• Ask students to identify certain, uncertain and impossible out comes • Ask students to identify experiment, trial, event and sample space • Ask students to solve real-life applications of example of certain and impossible ou comes. • Provide descriptive feedbac for learners to improve thei learning
• Find the probability of simple events	**8. 2 Probability of Simple Events *(8 periods)***	• From the above experiment, students discuss to derive the formula of the probability of a simple event as: P (Event) = $\frac{Number\ of\ times\ the\ event\ occur}{Total\ number\ of\ equally\ likely\ possible\ out\ comes}$ $= \frac{n(E)}{n(S)}$ • Assist students to determine the probabilities of simple events, **Example:** Two coins are tossed. What is the probability that both land heads up?	• Ask group of students to derive the probability of simple Events • Ask students to find the probability of simple events • Ask students to solve real-life applications of probability of simple events • Provide descriptive feedbac for learners to improve thei learning
		Because either coin can land heads up or tails up, the possible outcomes are as follows. HH = heads up on both coins HT = heads up on first coin and tails up on second coin TH = tails up on first coin and heads up on second coin TT = tails up on both coins So, the sample space is S = {*HH*, *HT*, *TH*, *TT*}. The event is E = {HH}. P (E) = $\frac{n(E)}{n(S)} = \frac{1}{4}$ • Assist students to give real-life examples of finding the probability of an event.	
• Solve the probability of real-life problems	**8.3 Applications on Business, Climate, Road Transport accidents, and Drug Effects *(4 periods)***	• Assist students to work in groups and communicate the results of the real-life applications in weather forecasting, insurance, car accident, drug usage by taking real data from the concerned offices.	• Ask group of students to solve the probability of real life application problems. • Observe students while they are working together • Ask your students to communicate their result • Provide descriptive feedbac for learners to improve thei learning